PG–456	PGDAM–11

P.G. DIPLOMA EXAMINATION — JUNE, 2018.

Applied Mathematics

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. What is sensitivity analysis in an linear programming problem? Discuss its significance fully.

2. A firm produces two products, say *X* and *Y*. Product *X* sells for a net profit of Rs. 80 per unit, while product *Y* sells for a net profit of Rs. 40 per unit. The goal of the firm is to earn Rs. 900 in the next week. Also, the management want to achieve sales volume for the two products close to 17 and 15 respectively. Formulate this problem as a good programming model.

3. State the 'principle of optimality' in dynamic programming and give a mathematical formulation of a dynamic programming problem.

4. Construct the network for the project whose activities are given below and determine the critical path and the project duration :

Activity :	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration (weeks) :	3	8	12	6	3	3	8	5	3	8

5. Solve the following problem graphically :

$$\text{Player A} \overset{\text{Player B}}{\begin{pmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{pmatrix}}$$

6. Describe the branch and bound method for the solution of integer programming problem.

7. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following :

   (a) the mean queue size

   (b) the probability that the queue size exceeds 10.

8. Solve the geometric programming problem :

   Minimize $f(x) = \frac{4}{9} x_1^{-1} x_2 x_2^{-\frac{1}{2}} + x_1^{-1} x_3^{-1}$

   Subject to the constraint :

   $\frac{2}{9} x_2^{\frac{1}{3}} x_3 + 5 x_1 x_2^{-\frac{2}{3}} = 1$; $x_i > 0$, $i = 1, 2, 3$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Solve the following linear programming problem by simplex method :

Maximize $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

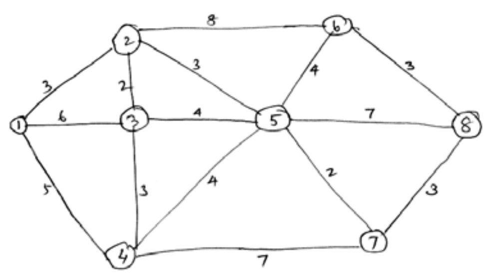
subject to
$$2x_1 + x_2 + 5x_3 + 6x_4 \le 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 \le 24$$
$$7x_1 + x_4 \le 70$$
and $x_1, x_2, x_3, x_4 \ge 0$

10. Use dual simplex method to solve the following L.P.P.

Minimize $z = 3x_1 + x_2$

subject to the constraints
$$x_1 + x_2 \ge 1,$$
$$2x_1 + 3x_2 \ge 2,$$
$$x_1, x_2 \ge 0$$

11. Consider the following network :



The distance (in miles) between different stations is shown on each link. Determine the shortest path from station 1 to station 8.

12. Explain dynamic programming and give a mathematical formulation of a dynamic programming problem.

13. Solve the following game by linear programming techniques :

$$\text{Player A}\overset{\text{Player B}}{\begin{pmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{pmatrix}}$$

14. Solve the following integer linear programming problem using the cutting-plane algorithm :

Maximize $z = 3x_1 + x_2 + 3x_3$

subject to the constraints : $-x_1 + 2x_2 + x_3 \le 4$

$$4x_2 - 3x_3 \le 2$$

$$x_1 - 3x_2 + 2x_3 \le 3$$

x_1, x_2 and x_3 all are non-negative integers.

15. For $(m/a/1)$ queuing model, derive the Pollaczek-Khintchine formula for expected number of customers in the system.

16. Use separable programming to solve the non-linear programming problem :

Maximize $f(x) = 3x_1 + 2x_2$

subject to the constraints

$g(x) = 4x_1^2 + x_2^2 \le 16$ and $x_1, x_2 \ge 0$.

PG–457	**PGDAM–12**

P.G. DIPLOMA EXAMINATION — JUNE 2018.

Applied Mathematics

GRAPH THEORY AND ALGORITHMS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that the sum of the degrees of the points of a graph G is twice the number of lines.

2. Prove that every connected graph has a spanning tree.

3. Prove that every block is 2-connected.

4. Prove that a partition $P = (d_1, d_2, ..., d_p)$ of an even number into p parts with $p-1 \geq d_1 \geq d_2, ... \geq d_p$ is graphical if and only if the modified partition. $p' = (d_2 - 1, d_3 - 1, ..., d_{d_1+1} - 1, d_{d_1+2}, ..., d_p)$ is graphical.

5. If G is a graph with $p \geq 3$ vertices and $\delta \geq \frac{p}{2}$ then prove that G is hamiltonian.

6. If G is a k-regular bipartite graph with $k. > 0$ then prove that G has a perfect matching.

7. Show that $\lambda^4 - 3\lambda^3 + 5\lambda^2 - 1$ cannot be the chromatic polynomial of a graph.

8. Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

PART B — $(5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. If G_1 is a (u_1, v_1) graph and G_2 a (u_2, v_2) graph, then prove that $G_1 \times G_2$ is a $(u_1\ u_2, v_1 u_2 + v_2 u_1)$ graph.

10. Let G be a (p, q) graph. Prove that the following statements are equivalent :

    (a) G is a tree.

    (b) Every two points of G are joined by a unique path.

    (c) G is connected and $p = q + 1$

    (d) G is acyclic and $p = q + 1$.

11. State and prove Merger's theorem.

12. State and prove Wang and Kleitman's theorem.

13. If G is evaluation then prove that Fleury's algorithm constructs an eulerian trial of G.

14. State and prove Hall's marriage theorem.

15. State and prove Vizing's theorem.

16. If G is a connected plane graph having V, E and F as the sets of vertices, edges and faces respectively. Then prove that $|V|-|E|+|F|=2$.

PG-458	**PGDAM-13**

P.G. DIPLOMA IN APPLIED MATHEMATICS EXAMINATION – JUNE 2018.

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. The contents of urns I, II and III are as follows:

   1 white, 2 black and 3 red balls

   2 white, 1 black and 1 red balls and

   4 white, 5 black and 3 red balls.

   one urn is chosen at random and two balls drawn from it. They happen to be white and red. What is the prove that they come from urns I, II or III?

2. A random variable X has the following probability functions:

Value of X, x:	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(a) Find k,

(b) Evaluate $P(X<6))$, $P(X \geq 6)$ and $P(0<X<5)$

(c) If $P(X \leq k) > \frac{1}{2}$ find the minimum value of k.

3. The probability density function of the random variable X follows the probability law:

$$p(x) = \frac{1}{2\theta} \exp\left(-\frac{|x-\theta|}{\theta}\right), -\infty < x < \infty.$$

Find m.g.f. of X. Hence or otherwise find $E(X)$ and $V(X)$.

4. State and prove Lindeberg – Levy theorem.

5. Define consistent estimation and prove that, of T_n is a consistent estimator of $\gamma(\theta)$ and $\psi[\gamma(\theta)]$ is a continuous function of $\gamma(\theta)$, then $\psi[T_n]$ is a consistent estimator of $\psi[\gamma(\theta)]$.

6. Given a random sample from a population with p.d.f. $f(x, \theta) = \frac{1}{\theta}$, $0 \leq x \leq \theta$.

Show that 100 $(1-\alpha)$% confidence interval for θ is given by R and R/ψ, where ψ is given by $\psi^{n-1}[n-(n-1)\psi] = \alpha$, and R is the sample range.

7. (a) Explain Likelihood ratio test.

(b) If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population $f(x, \theta) = \theta \exp(-\theta x), \quad 0 \leq x < \infty$. Obtain the values of type I and type II errors.

8. Let $X_1, X_2, X_3, ..., X_n$ be a random sample from cauchy population:

$$f(x, \theta) = \frac{1}{\pi} \frac{1}{1 + (x - \theta)^2}; \quad -\infty < x < \infty, \quad -\infty < \theta < \infty.$$

Examine if there exists a sufficient statistics for θ.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Support that two dimensional continuous random variable (X, Y) has joint p.d.f. given by

$$f(x, y) = \begin{cases} 6x^2 y, & 0 < x < 1, 0 < y < 1 \\ 0, & elsewhere \end{cases}$$

(a) Verify that $\int_0^1 \int_0^1 f(x, y)\, dx\, dy = 1$

(b) Find

$$P\left(0 < X < \frac{3}{4}, \frac{1}{3} < Y < 2\right),$$

$P(X+Y<1, P(X>Y))$ and $P(X<1/Y<2)$.

10. Derive Normal distribution as a limiting form of a Binomial distribution.

11. Let $X_1, X_2, ..., X_n$ be a random sample from a population with continuous density. Show that $Y_1 = \min(X_1, X_2, ..., X_n)$ is exponential with parameter $n\lambda$ if and only if each X_i is exponential with parameter λ.

12. State and prove Khinchin law of large numbers.

13. (a) If T is an unbiased estimator for θ, show that T^2 is a biased estimator for θ^2.

    (b) Prove that in sampling from a $N(\mu, \sigma^2)$ population, the sample mean is a consistent estimator of μ.

14. State and prove Rao-Blackwel theorem.

15. State and prove Neyman Pearson Lemma.

16. State and prove Cramer-Rao Inequality.

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