PG–391	**MMS-25**

M.Sc. DEGREE EXAMINATION — JUNE, 2018.

Second Year

Mathematics

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Let X be a topological space. Suppose that C is a collection of open set of X such that for each x in X and each open set U of X there is an element c of C such that $x \in C \subset U$, then prove that C is a basis for the topology of X.

2. Let Y be a subspace of a topological space. Show that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

3. Prove that a space X is locally path connected if and only if for every open set U of X, each path component of U is open in X.

4. Prove that every compact subspace of a Hausdorff space is closed.

5. Prove that a subspace of a Hausdorff space is Hausdorff and a product of Hausdorff space is Hausdorff.

6. State and prove Uniform Boundedness theorem.

7. If x and y are two vectors is a Hilbert space, then prove that $|(x, y)| \leq \|x\| \|y\|$.

8. If M is a linear subspace of a Hilbert space H, then prove that M is closed if and only if $M = M^{\perp\perp}$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that the topology on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.

10. State and prove uniform limit theorem.

11. Let X be a simply ordered set having the least upper bound property. Prove that in the order topology each closed interval in X is compact.

12. State and prove Lebesgue number lemma.

13. Prove that every regular space with a countable basis is normal.

14. State and prove open mapping theorem.

15. Let H be a Hilbert space and let $\{e_i\}$ be an orthonormal set in H. Prove that the following conditions are equivalent to one another :

(a) $\{e_i\}$ is complete.

(b) $x \perp \{e_i\} \Rightarrow x = 0$.

(c) If x is an arbitrary vector in H, then $x = \sum (x, e_i) e_i$.

(d) If x is an arbitrary vector in H, then $\| x \|^2 = \sum |(x, e_i)|^2$.

16. State and prove Polarization identity.

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M.Sc. DEGREE EXAMINATION — JUNE, 2018.

Second Year

Mathematics

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Write the dual of

$$\text{Min } Z = x_2 + 3x_3$$

$$\text{s.t. } 2x_1 + x_2 \le 3$$

$$x_1 + 2x_2 + 6x_3 \ge 5$$

$$-x_1 + x_2 + x_3 = 2$$

$$x_1, x_2, x_3 \ge 0.$$

2. Write the differences between CPM and PERT.

3. Solve the game

$$\begin{array}{ccc} & & \text{B} \\ & & \begin{array}{cc} \text{I} & \text{II} \end{array} \\ \text{A} & \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} 4 & 0 \\ 0 & 8 \end{pmatrix} \end{array}$$

4. Determine the saddle point and the value of the game.

	B_1	B_2	B_3	B_4
A_1	8	–2	9	–3
A_2	6	5	6	8
A_3	–2	4	–9	5

5. Explain the following terms:

(a) Jockeying

(b) Balking

(c) Reneging

(d) Forgetfulness property.

6. Write the Lagrange's necessary conditions for

Min $f(x) = x_1^2 + x_2^2 + x_3^2$

s.t. $g_1(x) = x_1 + x_2 + 3x_3 - 2 = 0$

$g_2(x) = 5x_1 + 2x_2 + x_3 - 5 = 0$

7. What do you mean by separable function? Give examples. Is Max $z = 4x_2$ separable?

8. Solve graphically

Max $z = 2x_1 + 3x_2$

s.t. $x_1 - x_2 \leq 2$

$x_1 + x_2 \geq 4$

$x_1, x_2 \geq 0.$

PART B — $(5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. Solve using simplex method.

Max $Z = 4x_1 + 10x_2$

s.t. $2x_1 + x_2 \le 50$

$2x_1 + 5x_2 \le 100$

$2x_1 + 3x_2 \le 90$

$x_1, x_2 \ge 0.$

10. Solve the primal using dual.

Min $Z = 2x_1 + 2x_2$

s.t. $2x_1 + 4x_2 \ge 1$

$-x_1 - 2x_2 \le -1$

$2x_1 + x_2 \ge 1$

$x_1, x_2 \ge 0.$

11. Draw the network for the following PERT network and find the critical path. What is the probability that the project will be completed in 27 days?

Activity:	1-2	1-3	1-4	2-5	2-6	3-6	4-7	5-7	6-7
Optimistic time (days):	3	2	6	2	5	3	3	1	2
Pessimistic time (days):	15	14	30	8	17	15	27	7	8
Most likely time (days):	6	5	12	5	11	6	9	4	5

12. Solve the game using simplex method.

$$B \overset{A}{\begin{pmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{pmatrix}}$$

13. Solve the following IPP by cutting plane method.

Min $Z = 10x_1 + 9x_2$

s.t. $x_1 \le 8$,

$x_2 \le 10$,

$5x_1 + 3x_2 \ge 45$

$x_1, x_2 \ge 0$, x_1 is an integer.

14. (a) Derive L_s and L_q for the model $(M/M/1)(G_D/\infty/\infty)$.

(b) Four counters are being run on the Frontier of a country to check the passports and necessary papers of the tourists. The tourist chooses a counter at random. If the arrival is Poisson at the rate λ and the service time is exponential with parameter $\lambda/2$, what is the steady state average queue at each counter?

15. Solve : Min $f(x) = x_1^2 + x_2^2 + x_3^2$

s.t. $x_1 + x_2 + 3x_3 - 2 = 0$, $5x_1 + 2x_2 + x_3 - 5 = 0$.

16. Solve the problem by separable programming algorithm:

Max $Z = x_1 + x_2^4$

s.t. $3x_1 + 2x_2^2 \le 9$, $x_1, x_2 \ge 0$.

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PG–393	**MMS–27**

M.Sc. DEGREE EXAMINATION — JUNE 2018.

Second Year

Mathematics

GRAPH THEORY AND ALGORITHMS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Write Dijkstra's algorithm.

2. If G is simple and $\delta \geq (p-1)/2$, then prove that G is connected.

3. Show that if e is k edge connected with $k > 0$ and $\in'$ is G set of k - edges of G, then prove that $w(G - E') \leq 2$.

4. State Menger's theorem.

5. Define 1-isomorphic graphs, 2-isomorphic graphs.

6. State Fleury's algorithm.

7. Show that if G is bipartite with $\delta > 0$, then G has $\delta -$edge colouring such that all δ colours are represented at each vertex.

8. Prove that k_5 is non-planar.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that a graph is bipartite if and only if it contains no odd cycle.

10. Write : (a) Kruskal's algorithm (b) Prim's algorithm.

11. Prove that for any graph G, $K(G) \le \lambda(G) \le \delta(G)$.

12. State and prove Hall's marriage problem.

13. Discuss relationship between Hamiltonian and Eulerian graphs.

14. State and prove Vizing's theorem.

15. In any graph G with $\delta > 0$ then prove that $\alpha' + \beta' = p$.

16. State and prove Euler's formula in planar graphs.

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PG–394	MMS-28

M.Sc. DEGREE EXAMINATION — JUNE, 2018.

Second Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours   Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let a_1, a_2 be constants and consider the differential equation $L(y) = y'' + a_1 y' + a_2 y = 0$. If r_1, r_2 are the distinct roots of the characteristic polynomial p where $p(r) = r^2 + a_1 r + a_2$ then prove that the function ϕ_1, ϕ_2 defined by $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = e^{r_2 x}$ are the solutions of the differential equation $L(y) = 0$.

2. Define Wronskian. Verify whether the functions $\phi_1(x) = e^x$ and $\phi_2(x) = e^{-x}$ are independent or not.

3. Prove that for any 'n', the coefficient of x^n in $P_n(x)$ is $\dfrac{(2n)!}{2^n(n!)^2}$.

4. Let $f(x)$ be periodic with period w. Let A be an $n \times n$ constant matrix. Then prove that a solution of $y' = Ay + f(x)$ is periodic of period w if and only if $y(0) = y(w)$.

5. Let f be continuous and satisfy a Lipschitz condition on a rectangle $R : |x - x_0| \leq a$, $|y - y_0| \leq b$ $(a, b > 0)$. If ϕ and φ are solutions of the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ on an interval I containing x_0, then prove that $\phi(x) = \varphi(x)$ for all $x \in I$.

6. Solve $\dfrac{\partial^4 z}{\partial x^4} + \dfrac{\partial^4 z}{\partial y^4} = \dfrac{\partial^4 z}{\partial x^2 \partial y^2}$.

7. Reduce the equation $\dfrac{\partial^2 z}{\partial x^2} + 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = 0$ into its canonical form.

8. Define :

   (a) Boundary value problem.

   (b) Interior and exterior Dirichlet problem.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let ϕ be any solution of $L(y) = y^{(n)} + a_1 y^{(n-1)} + a_2 y^{(n-2)} + \cdots + a_{n-1} y' + a_n y = 0$ on an interval I containing a point x_0. Then prove that $\| \phi(x_0) \| e^{-k|x-x_0|} \le \| \phi(x) \| \le \| \phi(x_0) \| e^{k|x-x_0|}$ for all x in I where $\| \phi(x) \| = \left(| \phi(x) |^2 + | \phi'(x) |^2 + \cdots + | \phi^{(n-1)}(x) |^2 \right)^{1/2}$ and $k = | a_1 | + | a_2 | + \cdots + | a_n |$.

10. Find the solution of the Initial value problem $y'' - 2y' + y = 2x$, $y(0) = 6$, $y'(0) = 2$.

11. Solve the Legendre equation using power-series technique.

12. Derive Bessel function of order 'n' of second kind.

13. Let $A(x)$ be an $n \times n$ matrix which is continuous on a closed and bounded interval. Then prove that there exists a solution to the initial value problem $y' = A(x) \cdot y$, $y(x_0) = x_0$, $(x_1, x_0 \in I)$ on I.

14. (a) Find a solution of $(D^2 - D')z = 2y - x^2$. (7)

(b) Classify the equation $u_{xx} + u_{yy} = u_{xy}$. (3)

15. Discuss the method of solving hyperbolic equations.

16. State and prove Kelvin's Inversion theorem.
