PG–378 **MMS–25**

M.Sc. DEGREE EXAMINATION –
DECEMBER, 2018.

Second Year

Mathematics

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a then basis for the topology of Y ,then prove that the collection $D = B \times C / B \in \mathcal{B}$ and $C \in \mathcal{C}$ is a basis for the topology of $X \times Y$.

2. Let $f : X \to Y$; let X be metrizable. Show that the function f is continues if and only if for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ converges to $f(x)$.

3. Prove that the continuous image of connected space is connected.

4. State and prove intermediate value theorem.

5. Prove that every metrizable space is normal.

6. State and prove closed graph theorem.

7. If M is a closed linear subspace of a Hilbert space H, then prove that $H = M \oplus M^{\perp}$.

8. Prove that an operator T on H is self adjoint if and only if (Tx, x) is real for all x.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let X be a topological space, show that the following conditions held:

   (a) ϕ and X are closed.

   (b) Arbitrary intersections of closed sets are closed

   (c) Finite unions of closed sets are closed.

10. If α is a linear continuum is the order topology then prove that α is connected and so intervals are rays in α.

11. Prove that the product of finitely many compact spaces is compact.

12. State and prove the Urysohn's Lemma.

13. State and Tietz extension theorem.

14. State and prove Hahn Banoch theorem.

15. Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

16. If $\{ e_i \}$ is an orthonormal set is on Hilbert space H, then prove that $\sum |(x, e_i)|^2 \leq \|x\|^2$ for every vector x is H.

PG–379	**MMS–26**

M.Sc. DEGREE EXAMINATION – DECEMBER, 2018.

Second Year

Mathematics

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Write the dual of the L.P.P.
   Minimize $Z = x_2 + 3x_3$
   Subject to $2x_1 + x_2 \leq 3$
   $x_1 + 2x_2 + 6x_3 \leq 5$
   $-x_1 + x_2 + x_3 = 2$
   $x_1, x_2, x_3 \geq 0$

2. Write minimal spanning tree algorithm.

3. Construct the network for the project whose activities and their relationship are as given below.
   Activities : A, D, E can start simultaneously
   Activities : B, C > A; G, F, > D, C : H > E, F

4. Solve the following 2×2 game.

$$\begin{array}{cc} & B \\ A & \begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix} \end{array}$$

5. Write the applications of integer programming.

6. State and prove Forget fullness property.

7. State and prove a necessary condition for X_0 to be an extreme point of $f(X)$.

8. Define :

   (a) Separable programming

   (b) Geometric programming.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Use simplex method to

   Minimize $Z = 4x_1 + 10x_2$

   Subject to $2x_1 + x_2 \leq 50$

   $2x_1 + 5x_2 \leq 100$

   $2x_1 + 3x_2 \leq 90$ and

   $x_1, x_2 \geq 0$

10. Compute the earliest start, earliest finish , latest start and latest finish of each activity of the project given below.

Activity :	1-2	1-3	2-4	2-5	3-4	4-5
Duration (in days)	8	4	10	2	5	3

11. By dynamic programming technique, solve the problem

Minimize $Z = x_1^2 + x_2^2 + x_3^2$

Subject to $x_1 + x_2 + x_3 \geq 15$ and
$x_1, x_2, x_3 \geq 0$

12. Solve the rectangular game whose payoff matrix for player A is $\begin{pmatrix} 2 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix}$.

13. Solve the following integer programming problem.

Maximize $Z = 2x_1 + 2x_2$

Subject to $5x_1 + 3x_2 \leq 8$
$2x_1 + 4x_2 \leq 8$
$x_1, x_2 \geq 0$ and are all integers.

14. Visitors parking at Ozark college is limited to only five spaces. Cars making use of this space arrive according to a Poisson distribution at the rate of six cars per hour. Parking time is exponentially distributed with a mean of 30 minutes visitors who cannot find an empty space immediately on arrival may temporarily wait inside the lot until a parked car leaves. That temporary space can hold only three cars. All other cars that cannot park or find a temporary waiting space must go elsewhere. Determine the following

    (a) The probability p_n of n cars being in the system.

    (b) The effective rate of which cars arrive at the lot.

    (c) the average number of cars in the lot.

15. Prove that the exponential distribution is based on three axioms.

16. Solve

    Minimize $Z = x_1^2 + x_2^2 + 5$

    Subject to $3x_1^4 + x_2 \le 243$

    $x_1 + 2x_2^2 \le 32$

    $x_1, x_2 \ge 0$

PG–380	**MMS–27**

M.Sc. DEGREE EXAMINATION —
DECEMBER 2018.

Second Year

Mathematics

GRAPH THEORY AND ALGORITHMS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Show that every cubic graph has an even number of vertices.

2. Prove that is a tree any two vertices are connected by a unique path.

3. Define a bipartite graph with examples.

4. Define connectivity and edge connectivity of a graph and illustrate with examples.

5. Write a note on 'Eulerian' graphs.

6. Prove that Hamiltonian graph is 2-connected.

7. Prove a non-empty graph G is 2-colourable if and only if G is bipartite.

8. Prove that graph K_5 is non-planar.

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. State and prove Cayley's formula.

10. Write the steps of Prim's algorithm.

11. For any Graph, prove $K(G) \leq \lambda(G) \leq \delta(G)$.

12. Explain in detail about Eulerian and Hamiltonian graphs with examples.

13. Explain "Marriage Problem".

14. State the prove Vizing theorem.

15. State and prove Brook's theorem.

16. Prove the Euler's formula for a connected plane graphs.

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PG–381	**MMS–28**

M.Sc. DEGREE EXAMINATION — DECEMBER, 2018.

Second Year

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let $\phi_1, \phi_2, \ldots, \phi_n$ be n–solutions of a linear differential equation $L(y) = 0$ on an interval I. Then prove that $\phi_1, \phi_2, \ldots, \phi_n$ are independent on I if $W(\phi_1, \phi_2, \ldots, \phi_n) \neq 0$ for all x in I.

2. Find a particular solution of $y'' + y = \text{cosec } x$.

3. Prove that for any 'n' the coefficient of x^n in $P_n(x)$ is $\dfrac{(2n)!}{2^n (n!)^2}$ where $P_n(x)$ is the Legendre polynomial.

4. Define singular point and regular singular point.

5. Let $A(x)$ be a continuous $n \times n$ matrix such that $A(x+w) = A(x)$, $w \neq 0$, $x \in (-\infty, \infty)$. Let $\Phi(x)$ be a fundamental matrix of $y' = Ay$. Then prove that $\Phi(x+w)$ is also a fundamental matrix of the system of equations.

6. Find a solution of $(D^2 - D^1)z = 2y - x^2$.

7. Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\, \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ into its canonical form.

8. Show that the one-parameter family of surfaces $x^2 + y^2 = cz^2$ can form a family of equipotential surfaces.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let ϕ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing x_0. Then prove that for all x in I, $\|\phi(x_0)\| e^{-K|x-x_0|} \leq \|\phi(x)\| \leq \|\phi(x_0)\| e^{K|x-x_0|}$ where $\|\phi(x)\| = \left(|\phi(x)|^2 + |\phi(x)|^2\right)^{1/2}$ and $K = |a_1| + |a_2|$.

10. Solve the IVP, $y'' - 2y' + y = 2x$, $y(0) = 6$, $y'(0) = 2$.

11. Prove that for each n there is one and only one polynomial solution $P_n(x)$ of degree-n for the Legendre equation $(1-x^2)y''-2xy'+n(n+1)y=0$, $P_n(1)=1$.

12. Derive Bessel's function of order 0 first kind.

13. Solve the IVP , $y''-2y'+y=0$, $y(0)=0$, $y'(0)=1$ on $[0, a]$ where a is any positive real number.

14. Let $A(x)$ be an $n \times n$ matrix which is continuous on an internal I. Suppose a matrix satisfies $Y' = A(x) \cdot Y$, $x \in I$, then prove that $(\det \Phi)' = (\text{trace } A)$ Let Φ and equivalently $\det \Phi(x_0)\ e \int_{x_0}^{x} \text{trace } A(s)\, ds$.

15. (a) Classify the equations :

    (i) $u_{xx} + u_{yy} = u_{xy}$

    (ii) $u_{xx} + u_{yy} = u_{zz}$

    (iii) $u_{xx} + u_{yy} + u_{zz} = 0$. (2 + 1 + 1)

    (b) Find a particular integral of $(D^2 - D^1)z = e^{x+y}$. (6)

16. State and prove Kelvin's inversion theorem.