PG–387	**MMS–15/ PGDMAT–11**

M.Sc. DEGREE/P.G. DIPLOMA EXAMINATION – JUNE, 2018.

First Year

Mathematics

ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let G be a group in which $(ab)^m = a^m b^m$ for three consecutive integers and for all $a, b \in G$. Prove that G is abelian.

2. Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

3. Prove that a finite integral domain is a field.

4. If U, V are ideals of R, let $U+V = \{u+v : u \in U, v \in V\}$. Prove that $U+V$ is also an ideal of R.

5. If V is a finite-dimensional space over F, prove that any two bases of V, have the same number of elements.

6. If V is a vector space and $u, v \in V$, then prove that $|(u,v)| \leq \|u\| \|v\|$.

7. If L is a algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

8. If V is finite-dimensional over F, prove that $T \in A(V)$ is regular if and only if T maps V onto V.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. State and prove first part of Sylow's theorem.

10. State and prove Cayley's theorem.

11. If R is a ring with unit element, then for all $a, b \in R$ prove that

(a) $a.0 = 0.a = 0$

(b) $a(-b) = (-a)b = -(ab)$

(c) $(-a)(-b) = ab$

(d) $(-1)a = -a$

(e) $(-1)(-1) = 1$.

12. Prove that every integral domain can be imbedded is a field.

13. If $v_1, v_2, ..., v_n$ is a basis of V over F and if $w_1, w_2, ... w_m$ in V are linearly independent over F, prove that $m \leq n$.

14. If V and W are of dimensions m and n respectively over F, then prove that $Hom(V, W)$ is of dimensions mn over F.

15. If F is of characteristic 0 and if a, b are algebraic over F, then prove that there exist an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.

16. If $T \in A(V)$ has all its characteristic roots is F, then prove that there is a basis of V is which the matrix of T is triangular.

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PG–388	**MMS–16/ PGDMAT–12**

M.Sc. DEGREE/P.G. DIPLOMA EXAMINATION – JUNE 2018.

First Year

Mathematics

REAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Prove that the ordered set R has the least upper bound property.

2. Prove that compact subsets of metric spaces are closed.

3. Prove that if f is continuous at a point $p \in E$ and if g is continuous at $f(p)$ then prove that $h = g \circ f$ is continuous at p.

4. Let f be monotonic on (a, b), then prove that the set of pints of (a, b) at which f is discontinuous is almost countable.

5. State and prove mean value theorem.

6. If p^* is a refinement of p then prove that $U(p^*, f, \alpha) \leq U(p, f, \alpha)$.

7. State and prove Weierstrass theorem.

8. Prove that a linear operator A on R^n is invertible if and only if $\det[A] = 0$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that for every real $x > 0$ and every integer $n > 0$ there is one and only one real y such that $y^n = x$.

10. Prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$.

11. Let f be a continuous mapping of a compact metric space X in to a metric space Y, then prove that f is uniformly continuous on X.

12. State and prove L'Hospital rule.

13. If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\Lambda(r) = \int_r^b |\gamma'(t)| \, dt$.

14. State and prove the Stone-Weierstrass theorem.

15. State and prove Parseval's theorem.

16. State and prove the contraction principle.

PG–389	**MMS–17**

M.Sc. DEGREE EXAMINATION — JUNE, 2018.

First Year

Mathematics

COMPLEX ANALYSIS AND NUMERICAL ANALYSIS

Time : 3 hours | Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that real and imaginary parts of an analytic function satisfies Laplace equation.

2. Find the bilinear transformation which maps the points $Z = -i, 0, i$ into the points $W = -1, i, 1$ respectively.

3. Find the Taylor's Expansion for the function $f(z) = \dfrac{(z-2)(z+2)}{(z+1)(z+4)}$ for $|z| < 1$.

4. Prove that $\int_0^{2\pi} \frac{1}{a + b\cos\theta} d\theta = \frac{2\pi}{\sqrt{a^2 - b^2}}$ where $a > b > 0$.

5. Find the cube root of 24 by Newton Raphson method.

6. By Gauss Elimination find A^{-1} of $A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$.

7. Find the first derivative of y at $x = 1.5$ from the following data

x:	1.5	2.0	2.5	3.0	3.5	4.0
y:	3.375	7.0	13.625	24.0	38.875	59.0

8. Using Taylor's method, complete $y(1.1)$, $y(1.2)$ correct to four decimal places given $y' = x + y;\ y(1) = 0$.

SECTION B — $(5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. Derive C-R equations in Cartesian Co-ordinates.

10. Discuss the transformation $W = \sin z$.

11. State and prove maximum modulus theorem.

12. Find the number of zeros of the function $f(z) = z^7 - 4z^5 + z^2 - 1$ which lie inside the circle $C : |z| = 1$.

13. Using Newton's method, find a root between 0 and 1 of $x^3 = 6x - 4$ correct to three decimal places.

14. From the following table, estimate $e^{0.644}$ correct to five decimals using Stirling's formula.

x:	0.61	0.62	0.63	0.64
e^x:	1.840431	1.858928	1.877610	1.896481
x:	0.65	0.66	0.67	
e^x:	1.915541	1.934792	1.954237	

15. Using Lagrange's formula find $f(z)$ from the following table:

x:	0	1	3	4
$f(x)$:	–12	0	6	12

16. Given $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 2$, $y(0.2) = 2.0933$, $y(0.4) = 2.1755, y(0.6) = 2.2493$. Find $y(0.8)$ using Milne's predictor-corrector method.

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PG–390	MMS–18

M.Sc. DEGREE EXAMINATION — JUNE, 2018.

First Year

Mathematics

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let X have the probability density function $f(x) = \begin{cases} \dfrac{x+1}{2}, & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$. Find the mean and variance.

2. Let X have the probability density function $f(x) = \begin{cases} \dfrac{x^2}{9}, & 0 < x < 3 \\ 0, & \text{elsewhere} \end{cases}$. Find the probability density function of $Y = X^3$.

3. Let X equal the length of life of 60 watt light bulb marketed by a certain manufacturer of light bulbs. Assume that the distribution of X is $N(\mu, 1296)$. If a random sample of $n = 27$ bulbs were tested until they burned out, yielding a sample mean of $\overline{X} = 1478$ hrs. Find the 95% confidence interval for μ.

4. Let X be a random variable having the probability density function $f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$.
Test the simple hypothesis $H_0 : \theta = 2$ against the alternative simple hypothesis $H_1 : \theta = 4$, use a random sample X_1, X_2 of size $n = 2$ and define the critical region $C = \{(x_1, x_2) / 9.5 \leq x_1 + x_2 < \infty\}$. Also find the power of the test.

5. Let $X_1, X_2, \ldots X_n$ denote a random sample from a poisson distribution, that has the mean $\theta > 0$. Prove that $\overline{X}$ is an efficient estimator of θ.

6. Find $P_r\left(0 < X_1 < 1/3, 0 < X_2 < 1/3\right)$, if the random variables X_1 and X_2 have the joint probability density function

$$f(x_1, x_2) = \begin{cases} 4x_1(1 - x_2), & 0 < x_1 < 1, \\ & 0 < x_2 < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

7. Let $\overline{X}$ denote the mean of a random sample of size 128 from a gamma distribution with $\alpha = 2$ and $\beta = 4$. Find $P_r(7 < \overline{X} < 9)$.

8. Let $X_1, X_2, ... X_n$ denote the observations of a random sample of size $n > 1$ from a distribution that is $b(1, \theta), 0 < \theta < 1$. Let $Y = \sum_{i=1}^{n} X_i$. Find the unbiased and minimum variance estimator of Y / n.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Find the moment generating function of Gamma distribution and also find its mean and variance.

10. Let X_1, X_2 be a random sample from the normal distribution $n(0,1)$. Show that the marginal probability density function of $Y = \frac{X_1}{X_2}$ is the Cauchy probability density function.

11. Let two random samples, each of size 10, from two independent normal distributions $n(\mu_1, \sigma_1^2)$ and $n(\mu_2, \sigma_2^2)$ yield $\overline{x} = 4.8, s_1^2 = 8.64$, $\overline{y} = 5.6$, $s_2^2 = 7.88$. Find a 95% confidence interval for $\mu_1 - \mu_2$.

12. State and prove Rao-Blackwell theorem.

13. Let X has a probability density function

$$f(x,\theta) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0,1 \\ 0, & \text{elsewhere} \end{cases}.$$ Using sequentially probability ratio test, test the hypothesis $H_0 : \theta = 1/3$ and $H_1 : \theta = 2/3$.

14. Let X and Y have bivariate normal distribution with parameters $\mu_1 = 3$, $\mu_2 = 1, \sigma_1^2 = 16, \sigma_2^2 = 25$ and $p = 3/5$. Determine the following.

(a) $P_r(3 < Y < 8)$

(b) $P_r(3 < Y < 8 / X = 7)$

(c) $P_r(-3 < X < 3)$

(d) $P_r(-3 < X < 3 / Y = -4)$.

15. Let Z_n be $\psi^2(n)$. Find the limiting distribution of the random variable $Y_n = \dfrac{Z_n - n}{\sqrt{2n}}$.

16. Let $X_1, X_2, X_3, ,, X_n$ be a random sample from the exponential distribution with probability density function $f(x,\theta) = 1/\theta\, e^{-x/\theta}, 0 < x < \infty$, $\theta \in \Omega = \{\theta / 0 < \theta < \infty\}$. Find the maximum likelihood estimator for θ.
