PG–376	**MMS–17**

M.Sc. DEGREE EXAMINATION – DECEMBER, 2018.

First Year

Mathematics

COMPLEX ANALYSIS AND NUMERICAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that $w = |z|^2$ is continuous but nowhere differentiable except at $z = 0$.

2. Show that the product of two bilinear transformations is a bilinear transformation.

3. Using Cauchy integral formula, evaluate $\int_C \frac{zdz}{(9-z^2)(z+i)}$, where C is the circle $|z| = 2$ described is positive sense.

4. State and prove Morera's theorem.

5. Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three decimal places.

6. The area A of a circle of diameter d is given for the following values :

d :	80	85	90	95	100
A :	5026	5674	6362	7088	7854

Find approximate values for the area of cycle of diameter 82.

7. Calculate $\int_0^{\pi/2} e^{\sin x} dx$ correct to four decimal places by using Simpson's 3/8 rule.

8. Use Picard's method to approximate y when $x = 0.1, 0.2$ given that $y = 0$ when $x = 0$, $\frac{dy}{dx} = x + y$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Discuss the transformation $w = z^2$.

10. State and prove Cauchy's integral theorem.

11. Evaluate $\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}$, $a^2 < 1$.

12. Find the root of $x^2 - 5x + 2 = 0$ correct the five decimal places by Newton-Raphson method.

13. Apply Gauss-Jordan method to find the inverse of $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$.

14. Use Stirling's formula to find y_{28}, when

$y_{20} = 49225$; $y_{25} = 48316$; $y_{30} = 47236$
$y_{35} = 45926$; $y_{40} = 44306$.

15. Find the value of the double integral $I = \int_2^{3.2} \int_1^{2.8} \frac{1}{x + y} dy\, dx$.

16. Given $\frac{dy}{dx} = x^2(1 + y)$ and $y(1) = 1$; $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.974$, evaluate $y(1.4)$ by Adams-Bashforth method.

PG–377	**MMS–18**

M.Sc. DEGREE EXAMINATION — DECEMBER 2018.

First Year

Mathematics

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. A machine consists of two parts A and B. In a sample of 100 items in Part – A, there are 5 defectives and in a sample of 100 items in Part – B, there are 10 defectives. One item is chosen from each of these samples and the machine is composed. What is the probability that the machine composed in non–defective?

2. A random variable X has the following probability function.

X :	0	1	2	3	4	5	6	7
P(X) :	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

(a) Find K

(b) Evaluate $P(X < 6)$

(c) If $P(X \leq a) > \frac{1}{2}$. Find the minimum value of a.

3. State the properties of t – distribution.

4. Find the MGF of the Poisson distribution and hence obtain its mean and variance.

5. A sample of 400 male students is found to have a mean height of 171.38cms. Can it be reasonably regarded as a sample from a large population with mean height 171.17cms and standard deviation of 3.30cms at 5% level of significance?

6. If $X_1, X_2, \ldots\ldots, X_n$ is a random sample from the Poisson distribution with parameter λ, obtain the M.L.E. of λ.

7. State the properties of maximum likelihood estimator.

8. Define the following :

   (a) Unbiased estimator

   (b) Consistant estimator

   (c) Sufficient estimator

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. State and Prove Bayes theorem.

10. State and Prove Chebychev inequality.

11. State and Prove the central limit theorem.

12. Show that $\frac{ns^2}{n-1}$ is a consistent estimator of σ^2.

13. Find the confidence interval for the difference between the means when the variance of both the population are known.

14. Let X have a p.d.f. of the form $f(x, \theta) = \theta x^{\theta - 1}$, $0 < x < 1$, zero elsewhere, where $\theta \in \{\theta : \theta = 1, 2\}$. To test the simple hypothesis $H_o : \theta = 1$ against the alternative, simple hypothesis $H_1 : \theta = 2$, use a random sample X_1, X_2 of size $n = 2$, find the critical region to be $c = \left\{(x_1, x_2); \frac{3}{4} \leq x_1 \, x_2\right\}$. Find the power function of the test.

15. State and Prove Neyman Pearson theorem.

16. State and Prove Rao Cramer inequality.

PG–374	**MMS–15**

M.Sc. DEGREE EXAMINATION – DECEMBER, 2018.

First Year

Mathematics

ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Show that every subgroup of cyclic group is cyclic.

2. Let G be the internal object product of its normal subgroups $N_1, N_2, ..., N_n$. Then prove that for $i \neq j$, $N_i \cap N_j = (e)$ and $ab = ba$ for $a \in N_i$ and $b \in N_j$.

3. Define an ideal of a ring R and show that the kernel of a homomorphism of a ring R into a ring R' is an ideal of R.

4. Let a and b be two non zero elements of an Euclidian ring R. If b is not unit in R, show that $d(a) < d(ab)$.

5. Prove that L(S) is a subspace of V.

6. If V is a finite dimensional over F, then prove that for $S, T \in A(V)$

   (a) $r(ST) \leq r(T)$

   (b) $r(TS) \leq r(T)$.

7. State and prove remainder theorem.

8. If $T \in A(V)$ is nilpotent, then prove that $\alpha_0 + \alpha_1 T + ... + \alpha_n T^n$, where $\alpha_i \in F$ is invertible if $\alpha_0 \neq 0$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If H and K are finite subgroups of a group G, show that $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$.

10. Prove that every finite abelian group is the direct product of cyclic groups.

11. Let R be a commutative ring with unity and M is an ideal of R. Show that M is a maximal ideal of R if and only if R/M is a field.

12. State and prove unique factorization theorem on Euclidean rings.

13. If $v_1, v_2, \ldots\ldots, v_n$ are in V, then prove that either they are linearly independent or some v_i is a linear combination of the preceding ones $v_1, v_2, \ldots., v_{i-1}$.

14. Prove that the number e is transcendental.

15. State and prove fundamental theorem of Galois theory.

16. If $W \subset V$ is invariant under T, then prove that T induces a linear transformation $\overline{T}$ on V/W defined by $(v + W)T = vT + W$. If T satisfies the polynomial $\gamma(x) \in F(x)$, then prove that $\overline{T}$ also satisfies the polynomial $\gamma(x) \in F(x)$. If $p(x)$ is the minimal polynomial for $\overline{T}$ over F and if $p(x)$ is that for T then prove that $p_1(x) | p(x)$.

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PG–375	MMS–16

M.Sc. DEGREE EXAMINATION – DECEMBER, 2018.

First Year

Mathematics

REAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. State and prove Schwarz inequality.

2. Prove that the set E is open if and only if its compliment is closed.

3. Prove that a mapping *f* of a metric space X in to a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

4. Let *f* be defined on [*a, b*]; if *f* has a local maximum at a point $x \in (a, b)$ and if $f'(x)$ exists then prove that $f'(x) = 0$.

5. If P* is a refinement of P, then prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$.

6. State and prove Cauchy criterion for uniform convergence.

7. Establish the relation between Gamma function and beta function.

8. Prove that a linear operator A on a finite dimensional vector space X is one-to-one if and only if the range of A is all of X.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that every k-cell is compact.

10. State and prove root test.

11. Let f be a continuous mapping of a compact metric space X in to a metric space Y, then prove that f is uniformly continuous on X.

12. State and prove Taylor's theorem.

13. Prove that $f \in R(\alpha)$ on $[a,b]$ if and only if for every $\varepsilon > 0$ there exist a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.

14. Prove that there exists a real continuous function on the real line is nowhere differentiable.

15. Derive Stirling's formula.

16. State and prove inverse function theorem.
