TAMIL NADU OPEN UNIVERSITY

Chennai - 15

School of Science

ASSIGNMENT

Programme Code No	: 231
Programme Name	: M.Sc., Mathematics
Course Code & Name	: MMS-15, Algebra
Batch	: AY 2018-19
No.of Assignment	: One Assignment for Each 2 Credits
Maximum Marks	: 100
Weightage	: 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. If G is a finite group and $a \in G$ then prove that $a^{o(G)}$ = e, proving Lagrange's theorem.
2. Prove that every integral domain can be imbedded in a field.
3. If L is a finite extension of K and if K is a finite extension of F, then L is a finite extension of F. Moreover, [L : F] = [L : K] [K : F]
4. Prove that if V is finite dimensional over F then V is isomorphic to $F^{(n)}$ where n is the number of elements in any basis of V over F.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. State and prove Sylow's Theorem
2. State and prove Unique Factorization Theorem on Euclidean ring.
3. If K is a finite extension of F, then G(K:F) is a finite group and its order, O(G(K:Q)) satisfies O(G(K:F)) ≤ [K:F].

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove Cauchy's theorem on Group.
2. State and prove Unique Factorization Theorem.
3. Prove that an element $a \in K$ is algebraic over F iff F(a) is a finite extension of F.
4. If V and W are of dimensions m and n respectively over F, then prove that Hom (V,W) is of dimension mn over F.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Prove that every finite abelian group is the direct product of cyclic groups.
2. Prove that V is finite – dimensional over F, then any two bases of V have the same number of elements.
3. Prove that K is a normal extension of F iff K is the splitting field of some polynomial over F.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that if G is a finite group, and p is a prime number with $p^n/o(G)$ and $p^{n+1} \nmid O(G)$, then any two subgroups of G of order p^n are conjugate.
2. Show that the set of Gaussian integers is an Euclidean domain.
3. Prove that F is of characteristic 0 and if *a, b* algebraic over F, then there exists an element c $\in F(a,b)$ such that $F(a,b) = F(c)$.
4. Let V be a finite dimensional inner product space, then prove that V has an orthonormal set as basis.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Prove that Two abelian groups of order p^n are isomorphic iff they have the same invariants.
2. If A and B are finite dimensional subspaces of a vector space V, then prove that A + B is finite dimensional and dim (A + B) = dim A + dim B – dim (A∩B).
3. If T ∈ A (V) has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that the number of p – sylow subgroups in G, for a given prime, is of the form 1 + kp.
2. Let R be a Euclidean ring, then any finitely generated *R* – module *M* is the direct sum of a finite number of cyclic sub modules – Prove.
3. Let F be a field which has all the n^{th} roots of unity, for every integer. If $p\ (x) \in F\ [x]$ is solvable by radicals for F, then prove that the Galois group of $p\ (x)$ is solvable group.
4. If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. R is a commutative ring with unit element and M is an ideal of R. Prove that M is a maximal ideal of R iff R/M is a field.
2. Show that the number e is transcendental.
3. Prove that two nilpotent linear transformations are similar if and only if they have the same invariants.

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ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-16, Real Analysis
Batch : AY 2018-19
No.of Assignment : One Assignment for Each 2 Credits
Maximum Marks : 100
Weightage : 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Define a compact set, given example and prove that closed subsets of compact sets are compact.
2. State and prove chain rule differentiation.
3. State and prove Cauchy criterion for uniform convergence on functions.
4. List properties of Gamma function.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Prove that every k-cell in R^k is compact.
2. If f is monotonic on [a,b] and α is monotonically increasing and continuous function on [a,b] with α(a) and α (b) finite, then prove that f$\in R(\alpha)$ on [a,b].
3. State and prove Inverse function theorem.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that Cantor set is uncountable.
2. State and prove Taylor's Theorem.
3. State and prove Weierstrass M-test for uniform convergence of series of functions.
4. Write a note on Beeta function.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. State and prove Heine Borel theorem
2. Write a note on Rectifiable curves.
3. State and prove Implicit function Theorem.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that continuous image of compact subset is compact.
2. Prove that continuous functions are Riemann-Stieltjes integrable.
3. Write a note on Equi continuous family of functions.
4. State and prove Approximation theorem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. State and prove Riemann's Theorem on rearrangement.
2. State and prove Stone weierstrass theorem.
3. State and prove Rank theorem.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that continuous image of connected subset is connected.
2. Prove that product of integrable functions is integrable.
3. State and prove Parseval's theorem.
4. State and prove open mapping theorem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Define uniformly continuous function. Also prove that if f is a continuous mapping of a compact metric space X into a metric space Y, then f is uniformly continuous. Is compactness necessary in this result?
2. Prove that any polynominal with complex co-efficient has a root in C.
3. Define a contraction map and prove that if Ø is a contraction map on a complete metric space then prove that it has one and only fixed point.

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ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-17, Complex Analysis and Numerical Analysis
Batch : AY 2018-19
No.of Assignment : One Assignment for Each 2 Credits
Maximum Marks : 100
Weightage : 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Every harmonic function satisfies the complex form of the Laplace equation.
2. State and Prove Cauchy's integral formula for an annulus.
3. Explain Miline – Thomson method.
4. Use analytic method to find the approximate value of the root of the equation 3x -$\sqrt{1+\sin x}$ = 0.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Use Bisection method to find the approximate value of the root of the equation 3x -$\sqrt{1+\sin x}$ = 0.
2. Expand $\frac{z}{(z-1)(2-z)}$ in Laurent series valid for i) $|z| < 1$, ii) 1< $|z| < 2$, iii) $|z| > 2$ iv) $|z-1| > 1$, (v) 0 < $|z-2|$ < 1.
3. Solve : y' = xy, y(1) = 2, for x = 1.4 using Runge-Kutta method.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove Lucas theorem.
2. State and prove cauchy's theorem for a rectangle.
3. State and prove Weierstrass theorem on essential singularity.
4. Find a real root of the equation $x^3 - 2x - 5 = 0$ using Regula falsi method.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Explain Gauss elimination method and solve the following system of equations by Gauss elimination method.

   $10x_1 + x_2 + x_3 = 12$, $x_1 + 10x_2 + x_3 = 12$, $x_1 + x_2 + 10x_3 = 12$.

2. Solve the system of equations by Gauss-Seidel method.

   $8x - y + z = 18$, $2x + 5y - 2z = 3$, $x + y - 3z = -6$.

3. Using modified Euler method, find the value of y when x = 0.3, given that $\frac{dy}{dx} = x + y$, $y(0)=1$.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove General form of Cauchy's theorem
2. State and prove Residue theorem.
3. State and prove the maximum principle.
4. Applying Newton-Raphson method find the real root of $x^3 - 3x - 5 = 0$

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Explain Gauss - jordan method and solve the following system of equations by Gauss jordan method.

   $10x_1 + x_2 + x_3 = 12$, $x_1 + 10x_2 + x_3 = 12$, $x_1 + x_2 + 10x_3 = 12$.

2. Use power method to find the dominant eigen value and eigenvector of

   $$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

3. Explain Milne method and apply to find y(1.0) given that $y' = x-y^2$, $y(0) = 0$.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove Morera's theorem
2. State and prove Generalized argument principle.
3. Using trapezoidal rule with h = k = 0.5 and h = k = 0.25, evaluate $I = \int_1^2 \int_1^2 \frac{dx\,dy}{x+y}$
4. State and prove Rouche's Theorem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Solve the following equation using Guass-Jacobi iteration method.

   $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$.

2. Solve : $\frac{dy}{dx} = x + y$, $y(0) = 1$ by the Picard method of successive approximations.
3. Derive Simpson's one –third rule and Apply Simpson's one –third rule to evaluate the

   approximate value of the integral by dividing the range into 8 equal parts.

   $$\int_2^{10} \frac{dx}{1+x}$$

TAMIL NADU OPEN UNIVERSITY

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ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-18, Mathematical Statistics
Batch : AY 2018-19
No.of Assignment : One Assignment for Each 2 Credits
Maximum Marks : 100
Weightage : 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove Chebyshev's inequality.
2. State and prove Central limit theorem.
3. State and prove Rao – Cramer inequality.
4. A bowl contains 10 chips. Four of the chips are red, 5 are white, and one is blue. If three chips are taken at random and without replacement, compute the conditional probability that there is one chip of each colour relative to the hypothesis that there is exactly one red chip among the three.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Write a note on Conditional Probability.
2. Write a note on Normal Distribution.
3. Let Z_n be χ^2 (n). Find the limiting distribution of the random variable $y_n = \frac{z_n - n}{\sqrt{2n}}$.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove Baye's formula.
2. Let $\bar{X}$ denote the mean of a random sample of size 128 from a gamma distribution with α=2 and β = 4. Find Pr(7 < $\bar{X}$ < 9).
3. Explain Sequentially Probability Ratio Test.
4. Let F have an F – distribution with parameters r_1 and r_2 then $\frac{1}{F}$ has an F distribution with parameters r_2 and r_1.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Let f (x,y) = 2, 0 < x < y, 0 < y < 1, zero elsewhere, be the joint probability density function of X and Y. show that the conditional means are, respectively $\frac{1+x}{2}$, 0 < x < 1 and $\frac{y}{2}$, 0 < y < 1. show also that the correlation coefficient of X and Y is $\rho = \frac{1}{2}$.
2. Find the moment generating function of a bivariate normal distributions.
3. Let the random variable X have the probability density function f (x) = $\begin{cases} 1\ ; 0 < x < 1 \\ 0\ ; \text{elsewhere} \end{cases}$

   Let X_1 and X_2 denote a random sample from this distribution. Find the joint probability density function of Y_1 and Y_2 where $Y_1 = X_1 + X_2$, $Y_2 = X_1 - X_2$. Find also marginal probability density function of Y_1 and Y_2.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Find the moment generating function of binomial distribution and also find is mean and variance.
2. State and prove Neyman – Pearson theorem.

3. Let X_1, X_2, …, X_n denote a random sample from a Poisson distribution, that has the mean $\theta > 0$. Prove that $\bar{X}$ is an efficient estimator of θ.

4. Let $X_1, X_2, X_3, \ldots, X_n$ be mutually stochastically independent random variables that have, respectively, the Chi – square distributions of $r_1, r_2, r_3, \ldots, r_n$ degrees of freedom. Then the random variable $Y = X_1 + X_2 + X_3 + \ldots + X_n$ has a chi square distribution with $r_1 + r_2 + r_3 + \ldots + r_n$ degrees of freedom.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Write a note on Poisson Distribution.

2. Show that $Y = \frac{1}{1+\frac{r1}{r2}F}$ where F has an F distribution with parameters r_1 and r_2 has a beta distribution.

3. Let X_1, X_2, X_3 denote a random sample of size 3 from a distribution that is n(0,1). Find the probability density function of $Y = X_1^2 + X_2^2 + X_3^2$.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Let X and Y have a trinomial distribution. Find the correlation coefficient between X and Y.

2. State and prove Rao – Blackwell theorem.

3. Given that probability density function $f\ (x;\theta) = \frac{1}{\pi[1+(x-\theta)^2]}$, $-\infty < x, -\infty$, $-\infty < \theta, -\infty$. Show that the Rao – Cramer' lower bound is $\frac{2}{n}$, where n is the size of a random sample from this Cauchy distribution.

4. Let $X_1, X_2, X_3, \ldots, X_n$ denote a random sample of size n from a distribution that is n (μ, σ^2). Then the random variable $Y = \sum_{i=1}^{n} \left(\frac{X_i-\mu}{\sigma}\right)^2$ has a χ^2 distribution with n degrees of freedom.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Write a note on Binomial Distribution .
2. Write a note on Stochastic convergence.
3. Let $X_1, X_2, X_3, \ldots, X_n$ denote a random sample of size $n \geq 2$ from a distribution that is n (μ, σ^2). Let $\overline{X}$ and S^2 be the mean and variance of this random sample. Then

   (a) $\overline{X} \sim n\left(\mu, \frac{\sigma^2}{n}\right)$

   (b) $n\frac{S^2}{\sigma^2} \sim \chi^2\ (n-1)$ and

   (c) $\overline{X}$ and S^2 are stochastically independent.