TAMIL NADU OPEN UNIVERSITY

Chennai - 15

School of Science

ASSIGNMENT

Programme Code No : 131
Programme Name : B.Sc., Mathematics
Course Code & Name : BMS-11, Elements of Calculus
Batch : CY 2019
No.of Assignment : One Assignment for Each 2 Credits
Maximum Marks : 100
Weightage : 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Explain comparison test and show that the series $\frac{1}{log2} + \frac{1}{log3} + \ldots + \frac{1}{logn} + \ldots$ is divergent.
2. Find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the co-ordinate planes.
3. Find the radius of curvature of the cardiod r = $a(1 - \cos\theta)$.
4. Find the extreme values of $y^2 + 4xy + 3x^2 + x^3$.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Define Beta function and explain properties of Beta function.
2. Derive the reduction formula for $\int cos^n x\, dx$ and hence evaluate $\int_0^{\pi/2} cos^n x\, dx$.
3. State and prove Raabe's Test.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Test for convergence the series $\sum \frac{1}{n} \sin \frac{1}{n}$.
2. The cycloid $x = a\,(\theta - \sin\theta)$, $y = a\,(1 - \cos\theta)$ rotates about the tangent at its vertex. Find the surface area formed.
3. Find the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
4. Find the maximum and minimum values of $x - sin2x + \frac{1}{3}\, sin3x$ in $[-\pi, \pi]$.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. State and prove Cauchy's second theorem on limits.
2. Derive the reduction formula for $\int sin^n x\, dx$ and hence evaluate $\int_0^{\pi/2} sin^n x\, dx$.
3. State and prove D'Alembert's Ratio Test.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove Cauchy's general principle of convergence for series.
2. Prove that the perimeter of the Cardiod $r = a\,(1 - \cos\theta)$ is 8 a.
3. Find the envelope of the family of the curve $(x-a)^2 + (y-a)^2 = 4a$.
4. Find the n^{th} derivative of $x^3 \sin 2x$

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Derive the reduction formula for $\int cos^m x \cos nx \ dx$ and hence evaluate

   $\int_0^{\pi/2} cos^m x \cos nx \ dx$, and hence prove that $\int_0^{\pi/2} cos^n x \cos nx \ dx \ = \frac{\pi}{2^{n+1}}$

2. Derive the formula for Radius of curvature.
3. Define Gamma function, Show that the Gamma function $\Gamma(n)$ converges for n>0 and derive the recurrence formula.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that every convergent sequence is a Cauchy sequence. What about the converse? Justify.

2 Write a note on Jacobians

3. Find the area of the larger loop of the curve r = 2 + 4$\cos\theta$.

. 4. Prove that the envelope of a family of curves touches each member of the family.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks

1. Derive the reduction formula for $\int sin^m x \ cos^n \ x \ dx$ and hence evaluate

   $\int_0^{\pi/2} sin^m x \ cos^n \ x dx$, where m and n positive integers.

2. Prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$ can be transformd into $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{1}{r} \frac{\partial v}{\partial r}$ using polar coordinates.
3. State and prove Leibnitz Theorem and hence find the n^{th} derivative of e^x log *x*

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ASSIGNMENT

Programme Code No : 131
Programme Name : B.Sc., Mathematics
Course Code & Name : BMS-12, Trigonometry, Analytical Geometry (3d) and Vector Calculus
Batch : CY 2019
No.of Assignment : One Assignment for Each 2 Credits
Maximum Marks : 100
Weightage : 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Evaluate, by stoke's theorem $\int_C (e^x dx + 2y dy - dz)$ where C is the curve $x^2 + y^2 = 4$, $z = 2$.
2. If $F = x^2 y i + y^2 zj + z^2 xk$, find curl F and curl curl F.
3. Prove that the planes $5x - 3y + 4z = 1$, $8x + 3y + 5z = 4$ and $18x - 3y + 13z = 6$ contain a common line.
4. Find the angle between the planes $x - y + 2z - 9 = 0$ and $2x + y + z = 7$.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Verify Gauss's divergence Theorem for $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the rectangular parallelepiped $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.
2. Curl (u x v) = v ∇u -u ∇v + u div v – v div u.
3. (a) Derive the volume of a tetrahedron when the vertices are given.

   (b) Find the equation of the cone whose vertex is at the point (α, β, γ) and whose generators intersect the guiding curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, $z = 0$.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Verify Stoke's theorem for F = (2x – y) i – yz^2j – y^2zk where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary in the xy plane.
2. Derive the equation of the right circular cylinder whose radius is r and axis is the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$.
3. Find the equation of the plane through the line of intersection of the planes 2x – y + 5z - 3 = 0 and 4x + 2y –z + 7 = 0 and parallel to the z – axis.
4. Derive the equation of the plane in the Intercept form.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Verify Gauss's Divergence theorem for the function F = 2xzi + yzj + z^2k over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.
2. Prove Curl curl **F** = grad div F - $\nabla^2 F$.
3. (a) Derive the condition for two general spheres to cut orthogonally.

   (b) Show that the spheres $x^2 + y^2 + z^2 + 3x + 5y - z - 7 = 0$ and $x^2 + y^2 + z^2 + 2x - 7y - 3z - 6 = 0$ are orthogonal.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Show that the Green's theorem in a plane can be deduced as a special case of Stoke's theorem.

. 2. Find the equation of the cylinder whose generators are parallel to the line x = y = z and whose guiding curve is the circle $x^2 + y^2 + z^2 - 2x - 3 = 0, 2x + y + 2z = 0$.

3. Find the equation of the plane through the line of intersection of the planes x + y + z = 1, 2x + 3y + 4z – 7 = 0 and perpendicular to the plane x -5y + 3z = 5.
4. Sum the series : $\frac{\cos\alpha}{1!} + \frac{\cos 2\alpha}{2!} + \frac{\cos 3\alpha}{3!} + \ldots \infty$

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Verify Gauss's Divergence theorem over the cube bounded by the planes x = 0, x = 1; y = 0, y = 1; z = 0 and z = 1 for F = x^2 I + y^2 j + z^2 k.

2. (a) Find the equation of the cylinder whose generators intersect the curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, z = 0 and are parallel to line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

   (b) Find the equation of the right circular cylinder whose generators are parallel to the line x = -2y = 2z and which touch the sphere $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$.

3 (a). Find the Length of the Tangent from an external point to the general sphere

   (b) Find the condition that the plane $lx + my + nz = p$ may be a tangent plane to the Sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Verify the divergence theorem for F = 4xzi - $y^2j +$ yzk over the cube bounded by x = 0, x = 1, y = 1, z = 0, z = 1.
2. Derive the equation of the tangent plane to a sphere at a given point on it.
3. Find the equation of the plane through the point (1,-2,3) and the intersection of the planes 2x – y + 4z = 7 and x + 2y – 3z + 8 = 0.
4. Using Stoke's theorem evaluate $\int_c \ (yzdx + zxdy + xydz)$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks

1. Show that $\nabla^2 r^n = n(n + 1)\, r^{n-2}$.

2. (a) Find the equation of a cone with vertex at the origin.

   (b) Find the equation of the right circular cylinder of radius 2 whose axis passes through (1,2,3) and has direction cosines proportional to (2,-3,6).

3. (a). Find the equation of the right circular cone whose vertex is origin and guiding curve the circle $x^2 + y^2 + z^2 + 2x - y + 3z - 1 = 0, x - y + z + 4 = 0.$

   (b). Find the equation of the sphere having its centre (5,-2,3) and which touches the line $\frac{x-1}{6} = \frac{y+1}{2} = \frac{z-12}{-3}$.

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ASSIGNMENT

Programme Code No : 131
Programme Name : B.Sc., Mathematics
Course Code & Name : BMS-13, Differential Equations
Batch : CY 2019
No.of Assignment : One Assignment for Each 2 Credits
Maximum Marks : 100
Weightage : 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Solve : $(3y – 7x + 7)\, dx + (7y – 3x + 3)\, dy = 0$
2. Solve : $(y^4 + 2y)\, dx + (xy^3 + 2y^4 – 4x)\, dy=0$.
3. Solve : $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$.
4. Solve : $p(1 +q^2) = q(z-1)$.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Solve by the method of variation of parameters.

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^2e^x$$

2. Solve: $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \frac{\log x.sin(log\ x)+1}{X}$

3 (a). Solve : $(D^2 - 4D + 3)Y = \sin 3x \cos 2x$.

(b). Solve : $(D^2 – 2D + 4)\, Y = e^x \cos x$.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Solve : $(2x + 4y - 3)\,dy = (x + 2y - 3)\,dx$.
2. Solve : $(x^4 + y^4)\,dx - xy^3\,dy = 0$.
3. Solve : $\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{(x+y)^2}$.
4. Solve : $p + q = \sin x + \sin y$.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Solve by the method of variation of parameters.

   $\frac{d^2y}{dx^2} + 4y = \operatorname{cosec} 2x$

2. Solve: $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$
3. (a) Solve : $(D^2 - 8D + 9)Y = 8\cos 5x$.

   (b) Solve : $(D^2 - 5D + 6)\,Y = x^2 - x + 2$