UG–646	**BMS–11/ BMC–11**

M.A. DEGREE EXAMINATION – JUNE, 2018.

First Year

Mathematics

ELEMENTS OF CALCULUS

Time : 3 hours   Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Find the n^{th} differential co-efficient of $\cos x \cos 2x \cos 3x$.

2. Find the maximum value of function $f(x, y) = xy(a - x - y)$.

3. Find the radius of curvature at the point 't' of the curve $x = a(\cos t + t \sin t)$ $y = a(\sin t - t \cos t)$.

4. Evaluate $\int_0^{\pi/2} \sin^7 \theta \cos^5 \theta \, d\theta$.

5. Find the area of the cardiod $r = a(1 + \cos\theta)$.

6. If $\{s_n\}$ is a sequence of non-negative numbers and if $\lim_{n\to\infty} s_n = L$, then $L \geq 0$.

7. Prove $\lim_{n\to\infty} \frac{3n^2 - 6n}{5n^2 + 4} = \frac{3}{5}$.

8. Show that the series $\sum_{n=1}^{\infty} 2n/(n^2 - 4n + 7)$ diverges.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If $y = \left(x + \sqrt{1 + x^2}\right)^m$, prove that $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

10. (a) If $u = \frac{xy}{x + y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$.

(b) $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

11. Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

12. Prove that the radius of curvature at a point $(a\cos^3\theta, a\sin^3\theta)$ on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $3a\sin\theta\cos\theta$.

13. Find the length of one loop of the curve $3ay^2 = x(x-a)^2$.

14. Establish the reduction formula for $\int \sin^n x\,dx$ and hence evaluate $\int_0^{\pi/2} \sin^6 x\,dx$.

15. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.

16. If $\{s_n\}$ is a sequence of real number which converges to L then show that $\left\{s_n^2\right\}_{n=1}^{\infty}$ converges to L^2.

UG–647	**BMS–12/ BMC–12**

U.G. DEGREE EXAMINATION – JUNE 2018.

First Year

Mathematics

TRIGNOMETRY, ANALYTICAL GEOMETRY (3D) AND VECTOR CALCULUS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Express $\cos 6\theta$ interms of $\cos\theta$.

2. Prove that $\cos h^{-1} x = \log_e\left[x + \sqrt{x^2 - 1}\right]$.

3. Prove that the planes $x + 2y + 2z = 0$, $2x + y - 2z = 0$ are right angles.

4. Find the equation of the plane parallel to $2x - 3y + 5z + 12 = 0$ and passing through the points (2, 3, 1).

5. Find centre and radius of the sphere $16x^2 + 16y^2 + 16z^2 - 16x - 8y - 16z - 55 = 0$.

6. Find the equation of the sphere with centre (1, –1, 2) and touching the plane $2x - 2y + z = 3$.

7. If $\phi = x^2 + y^2 - z - 1$ find *grad* ϕ at (1, 0, 0).

8. If $\vec{F} = x^2\vec{i} + xy\vec{j}$ evaluate $\int_C \vec{F} \cdot d\vec{r}$ from (0, 0) to (1, 1) along the line $y = x$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that $\cos^8\theta = \frac{1}{2^7}[\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35]$.

10. Find $\log(1+i)$.

11. Find the equation of the plane passes through the intersection of the planes $2x + 3y + 10z - 8 = 0$ $2x - 3y + 7z - 2 = 0$ and is perpendicular to the plane $3x - 2y + 4z - 5 = 0$.

12. Find the image of the point (2, 3, 5) in the plane $2x + y - z + 2 = 0$.

13. Obtain the equation of the plane passing through the points (2, 2, –1), (3, 4, 2) and (7, 0, 6).

14. Find the shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-15}{-5}$.

15. Find the equation of the sphere which pass through the circle $x^2+y^2+z^2=5,\ x+2y+3z=5$ and touch the plane $4x+3y=15$.

16. Find $\nabla\cdot\vec{F}$ and $\nabla\times\vec{F}$ of the vector point function $\vec{F}=xz^3\vec{i}-2x^2yz\vec{j}+2yz^4\vec{k}$ at the point (1, –1, 1).

UG–648	**BMS–13/ BPHYA–01**

B.Sc. DEGREE EXAMINATION — JUNE, 2018.

First Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Solve : $\frac{dy}{dx} + y \cot x = \sin 2x$.

2. Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.

3. Solve : $\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$.

4. Solve : $p + q = \sin x + \sin y$.

5. Solve : $p^2z^2 + q^2 = 1$.

6. Find $L\left(\frac{e^{3t} - e^{-2t}}{t}\right)$.

7. Show that $L\left(e^{-at} f(t)\right) = F(s + a)$ where $F(s) = L(f(t))$.

8. Find the particular integral to $\left(D^2 - 4D - 12\right) y = \sin x \sin 2x$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Solve : $xy\, p^2 + (x + y)\, p + 1 = 0$.

10. Solve : $\left(D^2 - 4D + 3\right) y = x^3 e^{2x}$.

11. Solve by method of variation of parameters $\frac{d^2 y}{dx^2} + y = \operatorname{cosec} x$.

12. Solve $(mz - ny)\, dx + (nx - lz)\, dy + (ly - mx)\, dz = 0$.

13. Solve $\left(y^3 x - 2x^4\right) p + \left(2y^4 - x^3 y\right) q = qz\left(x^3 - y^3\right)$.

14. Solve $\frac{dx}{dt} = 2x - 3y$, $\frac{dy}{dt} = y - 2x$ using laplace transforms given that $x(0) = 8$, $y(0) = 3$.

15. Solve : $(p^2 + q^2)y = qz$.

16. Solve : $(x^2D^2 - 2xD - 4)y = x^2 + 2\log x$.
