PG-381	**MMS-25**

M.Sc. DEGREE EXAMINATION – JUNE 2019.

Second Year

Mathematics

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let Y be a subspace of X; let A be a subset of Y; let $\overline{A}$ denote the closure of A is X. Then prove that the closure of A is Y equals A$\cap$ Y.

2. State and prove the pasting lemma.

3. Prove that the union of a collection of connected sets that have a common point is connected.

4. Prove that every compact subset of a Hausdorff space is closed

5. Prove that every metrizable space is normal.

6. Prove that the operations of addition and scalar multiplication is normed linear space N are jointly continuous.

7. State and Prove Mukowski's inequality.

8. State and Prove parallelogram law of identity.

PART B — (5 × 10 = 50 marks)

Anser any FIVE questions.

9. Prove that the topologies on $\mathbb{R}^n$ induced by the euclidean metric d and the square metric P are the same as the Product topology on $\mathbb{R}^n$.

10. State and Prove the tube lemma.

11. State and Prove the Lebesgue number lemma.

12. State and Prove the Urysohn lemma.

13. State and Prove the Open mapping theorem.

14. State and Prove the Uniform boundedness theorem.

15. If M is a proper closed linear subspace of a Hilbert space H, then Prove that there exists a non-zero vector z_0 is H such that $z_0 \perp M$.

16. State and prove Riesz-Representation theorem.

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PG–382	MMS-26

M.Sc. DEGREE EXAMINATION – JUNE, 2019.

Second Year

Mathematics

OPERATIONS RESEARCH

Time : 3 hours — Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Define :
   (a) Objective function
   (b) Optimal solutions
   (c) Surplus variable.

2. Write the working procedure for dual simplex method.

3. Draw the network for the project whose activities and their procedure relationship are as given below :

Activities :	A	B	C	D	E	F	G	H	I
Immediate predecessor :	–	A	A	–	D	B, C, E	F	E	G, H

4. State Bellman's principle of optimality. Explain the forward and backward induction methods.

5. For what value of λ, the game with the following matrix is strictly determinable.

$$\text{Player A} \quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \text{Player B} \\ \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{pmatrix} \lambda & 6 & 2 \\ -1 & \lambda & -7 \\ -2 & 4 & \lambda \end{pmatrix} \end{array}$$

Ignoring the value of λ.

6. Explain the Branch and Bound method in an integer programming problem.

7. Babies are born is the sparsely populated state at the rate of one birth every 12 minutes.

   The time between births follows an exponential distribution. Find the following :

   (a) the average number of births per year

   (b) the probability that no birth.

8. Show how the following problem can be separable.

   Maximize $Z = x_1x_2 + x_3 + x_1x_3$

   Subject to

   $$\begin{aligned} & x_1x_2 + x_2 + x_1x_3 \le 10 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}.$$

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Solve by Simplex method

Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to

$x_1 + 4x_2 \leq 420$

$3x_1 + 2x_2 \leq 460$

$x_1 + 2x_2 + x_3 \leq 430$

10. Use dual simplex method to solve the LPP.

Min $Z = 2x_1 + x_2$

Subject to

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$x_1, x_2 \geq 0$

11. Write down the stepwise procedure for determining the critical path of a project.

12. Solve the following by dynamic programming problem :

Min $Z = y_1 + y_2 + \ldots + y_n$

Subject to

$y_1, y_2 \ldots y_n = b$

and

$y_1, y_2 \ldots y_n \geq 0$.

13. Solve the following game graphically.

Player B

$$\text{Player A}\begin{pmatrix} 1 & 6 & 4 & -1 \\ -1 & 1 & -2 & 5 \end{pmatrix}$$

14. Use Branch and Bound method to solve the following :

Max $Z = 2x_1 + 2x_2$

Subject to

$5x_1 + 3x_2 \le 8$

$x_1 + 2x_2 \le 4$

$x_1, x_2 \ge 0$ and integer

15. Discuss pure birth model.

16. Solve the following problem by geometric programming :

Minimize $z = 5x_1x_2^{-1} + 2x_1^{-1} + 5x_1 + x_2^{-1}$

$x_1, x_2 \ge 0$.

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PG-383	**MMS-27**

M.S.C. DEGREE EXAMINATION – JUNE 2019.

Mathematics

Second Year

GRAPH THEORY AND ALGORITHMS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Define an isomorphism between two graphs and give an example.

2. If G is a tree, then prove that q=p–1.

3. Show that each component of a forest is a tree.

4. Define connectivity and edge connectivity of a graph and illustrate with examples.

5. State and prove a necessary and sufficient condition for a graph to be eulerian.

6. Prove that Hamiltorian graph is 2-connected.

7. What is the importance of mycielski's construction.

8. Prove that the graph k3,3 is non-planar.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. What the steps of Dijkstra's Algorithm.

10. State and prove cayley's formula.

11. For any graph, prove that $K(G) \leq \lambda(G) \leq \delta(G)$.

12. Prove that a connected graph G is Euleriam if and only if each vertex of even degree.

13. Prove that every 3-regular graph without cut edges had a perfect matching.

14. Explain matching, matching number, M-alternating path, M-argumented path

15. State and prove vizing theorem.

16. State and prove Euler's formula in a connected plane graph.

PG–384	**MMS–28**

M.Sc. DEGREE EXAMINATION –
JUNE 2019.

Second Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Solve $y''-2y'-3y=0, y(0)=0\ y'(0)=4$.

2. Let $\phi_1\,\phi_2......\phi_n$ be n solutions of a linear differential equation $L(y)=0$ on an interval I. Then prove that if $\phi_1\,\phi_2......\phi_n$ are independent on I, then $W(\phi_1,\phi_2...\phi_n)\neq 0$ for all x in I.

3. Prove that $P_n(-x)=(-1)^n P_n(x)$ and hence $P_n(-1)=(-1)^n$ where $P_n(x)$ is the Legendre polynomial.

4. Prove that A solution matrix ϕ of $Y' = A(x)Y \ (x \in I)$ is a fundamental matrix if and only if $(\det \phi)(x) \neq 0$ for all $(x \in I)$.

5. Let $f(x)$ be periodic with period ω. Let A be an $n \times n$ constant matrix. Then prove that a solution of $y' = Ay + f(x)$ is periodic of periods ω if and only if $y(0) = y(\omega)$.

6. Solve $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = \frac{\partial^4 z}{\partial x^2 r y^2}$

7. Show that the one-parameter family of surfaces $x^2 + y^2 = cz^2$ can form a family of equipotential surfaces.

8. Define Interior and Exterior Neumann's problem.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that for any real number x_0 and constants α, β there exists a solution ϕ for the initial value problem $L(y) = y'' + a_1 y' + a_2 y = 0, y(x_0) = \alpha$ $y'(x_0) = \beta$ on $-\infty < x < \infty$.

10. Find the solution of the IvP

$y''-2y'+y=2x, y(0)=6,\ y'(0)=2.$

11. Define Legendre equation. Prove that

$$\int_{-1}^{1} P_n(x)\,P_m(x)=0 \text{ with } n\neq m.$$

12. Derive Bessel's function of order α of the first kind.

13. Let $A(x)$ be an $n\times n$ matrix which is continuous on an interval I. Then prove that the set of all solutions of the system of equations $y'=A(x)y$ forms an n-dimensional vector space over the field of complex numbers.

14. Reduce the equation $\frac{\partial^2 z}{\partial x^2}=x^2\frac{\partial^2 z}{\partial y^2}$ into its canonical form.

15. (a) Find a particular integral of $(D^2-D)\,z=e^{2x+y}$.

(b) Classify the equations:

(i) $u_{xx}+u_{yy}=u_{xy}$

(ii) $u_{xx}+u_{yy}=u_{zz}$ (4+6)

(iii) $u_{xx}+u_{yy}+u_{zz}=0.$

16. Let $y(x, y, z) = c$ be an one-parameter family of surfaces. If u is a family of equipotential surfaces, then prove that $\dfrac{\Delta^2 f}{\left(\left(\dfrac{\partial f}{\partial x}\right)^2 + \left(\dfrac{\partial f}{\partial y}\right)^2 + \left(\dfrac{\partial f}{\partial z}\right)^2\right)}$ is function of f alone.
