PG–443	**PGDAM–12**

P.G. DIPLOMA EXAMINATION – JUNE, 2019.

GRAPH THEORY AND ALGORITHMS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. If f is an isomorphism of the graph $G_1 = (V_1, X_1)$ to the graph $G_2 = (V_2, X_2)$ and $v \in V$, the prove that $\deg\ v = \deg f(v)$.

2. Draw all trees with 4 and 5 vertices.

3. Prove that there is no 3–connected graph with 7 edges.

4. Show that the sequence $(d_1, d_2, \ldots\ldots, d_p)$ is graphical iff the sequence $p-1-d_1, p-1-d_2, \ldots., p-1-dp$ is graphical.

5. Prove that the following statements are equivalent for a connected graph G.

   (a) G is eulerian

   (b) Every point of G has even degree

   (c) The set of edges of G can be partitioned into cycles.

6. Prove that K_5 is non-planar.

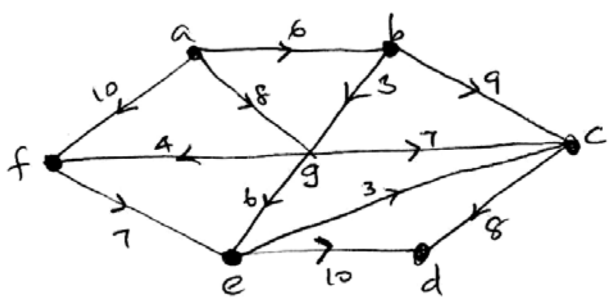
7. Prove that $\lambda^4 - 3\lambda^3 + 3\lambda^2$ cannot be the chromatic polynomial of any graph.

8. Define Geometric dual and show that, if G is planar, then every subgraph of G is planar.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Write Dijkstra's Algorithm and using this algorithm, find the distance between a and d from the following graph.



10. Write an algorithm to find a spanning tree and give an example.

11. State and prove Menger's theorem.

12. State and prove Wang and Kleitman's theorem.

13. Prove that a graph is Hamiltonian iff its closure is Hamiltonian.

14. State and prove Hall's marriage theorem.

15. State and prove Vizing's theorem.

16. If G is a connected plane graph having V, E and F as the sets of vertices, edges and faces respectively, then prove that $|V| - |E| + |F| = 2$.

PG–444	PGDAM–13

P.G. DEGREE EXAMINATION — JUNE, 2019.

MATHEMATICAL STATISTICS

Time : 3 hours   Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. State and prove Chebeshev's inequality.

2. Let X be a random variable with the following probability distribution :

x :	–3	6	9
$P(X = x)$:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

   Find $E(X)$ and $E(X^2)$ and using the laws of expectation. Evaluate $E(2X+1)^2$.

3. Let the random variable X assume the value 'r' with the probability law : $P(X = x) = q^{r-1}p$; $r = 1, 2, 3, \ldots$ Find the m.g.f of X and hence its mean and variance.

4. State and prove De'Moivre's theorem in central limit theorem.

5. Define unbiased estimator and if $x_1, x_2, \ldots, x_n$ are random observations on a Bernoulli variable X, taking the value 1 with probability θ and the value 0 with probability $1-\theta$, then show that $\frac{T(T-1)}{n(n-1)}$ is an unbiased estimate of θ^2 where $T = \sum_{i=1}^{n} x_i$.

6. Given one observations from a population with pdf :

$$f(x, \theta) = \frac{2}{\theta^2}(\theta - x), 0 \le x \le \theta.$$

Obtain $100(1-\alpha)\%$ confidence interval for θ.

7. (a) Define uniformly most powerful test.

(b) Let p be the probability that a coin will fall head in a single tons in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test.

8. (a) Define most efficient estimator

(b) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ.

(i) $t_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$

(ii) $t_2 = \frac{X_1 + X_2}{2} + X_3$

(iii) $t_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$

Where λ is such that t_3 is an unbiased estimator of μ? Find λ. Are t_1 and t_3 unbiased? State giving reasons, the estimator which is best among t_1, t_2 and t_3.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. The joint probability density function of a two dimensional random variable (X,Y) is given by :

$$f(x, y) = \begin{cases} 2, 0 < x < 1, 0 y < 1 \\ 0, \textit{elsewhere} \end{cases}$$

(a) Find the marginal density function of X and Y.

(b) Find the conditional density function of Y given $X = \lambda$ and conditional density function of X given $Y = y$.

(c) Check for independence of X and Y.

10. Derive Poisson distribution as a limiting form of a binomial distribution. Hence find β_1 and β_2 of the distribution.

11. Show that for a random sample of size 2 from $N(0, \sigma^2)$ population $E[X_{(1)}] = \frac{-\sigma}{\sqrt{\pi}}$.

12. State and prove Bernoulli's law of large numbers.

13. State and prove Rao-Blackwell theorem.

14. Let $\{T_n\}$ be a sequence of estimators such that for all $\theta \in \theta$ (a) $E_\theta(T_n) \to \gamma(\theta), n \to \infty$ and (b) $\text{var}_\theta(T_n) \to 0$ as $n \to \infty$. Then prove that T_n is a consistent estimator of $\gamma(\theta)$.

15. State and prove Newnan Pearson Lemma.

16. State and prove Rao-Cramer ineqality.