UG–328	**BMC–11/ BMS–11**

B.Sc. DEGREE EXAMINATION — JUNE, 2019.

First Year

Mathematics

ELEMENTS OF CALCULUS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. If $y = e^{a \sin^{-1} x}$ Prove that $(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 + a^2) = 0$.

2. Verify Euler's theorem for $u = ax^2 + 2hxy + by^2$.

3. Find the envelope of the family of lines $y = mx + \frac{a}{m}$ where 'a' is constant.

4. Evaluate $\int_0^{\pi/2} \sin^9 x \, dx$.

5. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

6. Evaluate $\int_0^{\pi/4} \log(1 + \tan\theta)\, d\theta$.

7. Show that $\lim_{n \to \infty} \frac{3n^2 + 2n + 5}{6n^2 + 4n + 7} = \frac{1}{2}$.

8. Discuss the convergence of the series

$\Sigma \frac{1}{\sqrt{n^3 + 1}}$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If $u = \log\left(x^2 + y^2 + z^2\right)$ prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{x^2 + y^2 + z^2}.$$

10. Prove that the envelope touches each member of the given family of curves at the corresponding points.

11. Derive the reduction formula for $\int \cos^n x\, dx$ and hence deduce $\int_0^{\pi/2} \cos^n x\, dx$.

12. Evaluate the double integral $\int_0^1 \int_0^{x^2} \left(x^2 + y^2\right) dy\, dx$.

13. Let $a_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$. Show that (a_n) diverges to α.

14. State and prove Cauchy's general principle of convergence.

15. Discuss the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + ...$

16. Verify Euler's theorem for the function $u = x^3 + y^3 + z^3 + 3xyz$.

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UG-329 BMS-12/BMC-12

U.G. DEGREE EXAMINATION – JUNE 2019.

First Year

Mathematics with Computer Applications

TRIGONOMETRY, ANALYTICAL GEOMETRY (3D) AND VECTOR CALCULUS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that

$$\cos 8\theta = 1 - 32 \sin^2 \theta + 160 \sin^2 \theta - 256 \sin^6 \theta + 128 \sin^8 \theta .$$

2. Sum to n terms of the series $\sin^3 \alpha + \sin^3 2\alpha + \sin^3 3\alpha + ...$

3. Find the equation of the line passing through $(3, 2, -1)$ and perpendicular to the plane $5x - 4y + 7z - 1 = 0$.

4. Find the centre and radius of the sphere

$$16x^2 + 16y^2 + 16z^2 - 16x - 8y - 16z - 55 = 0.$$

5. Find the equation of the sphere whose centre is $(1, -3, 4)$ and which passes through the point $(3, -1, 3)$.

6. If $\phi = x^2 + y - z - 1$ find grad ϕ at $(1, 0, 0)$.

7. If $u = x^2 + y^2$ prove that $\nabla^2 u = 0$.

8. State Stoke's theorem.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that

$$\cos^6\theta = \frac{1}{32}[\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10].$$

10. Separate into real and imaginary parts of $\tan(x + iy)$.

11. Show that the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x - 2y - z + 5 = 0$, $2x + 3y + 4z - 4 = 0$ are coplanar. Find their point of intersection.

12. Find the image of the line $\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-4}{2}$ in the plane $2x - y + z + 3 = 0$.

13. Find the equation of the sphere passing through the points $(1, 0, -1)$, $(2, 1, 0)$, $(1, 1, -1)$ and $(1, 1, 1)$.

14. Find divergence and curl of the vector point function $\vec{F} = x z^3 \vec{i} - 2x^2 y z \vec{j} + 2 y z^4 \vec{k}$ at the point $(1, -1, 1)$.

15. If $\vec{F} = (3x^2 + 6y)\vec{i} - 14 y z\vec{j} + 20 x z^2 \vec{k}$ evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the straight line joining $(0, 0, 1)$ to $(1, 1, 1)$.

16. Verify Green's theorem in the XY plane for $\int_C \{(xy + y^2)\, dx + x^2\, dy\}$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

UG–431	**BMS-13**

B.Sc. DEGREE EXAMINATION – JUNE, 2019.

First Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Solve : $y = xp + p^2$.

2. Solve : $(D^2 + 16)y = \cos 4x$.

3. Solve : $(x+z)^2 dy + y^2(dx + dz) = 0$.

4. Solve : $x^2 dx + y^2 dy = z(x+y)$.

5. Find : $L^{-1}\left[\frac{1}{(s-4)^5} + \frac{5}{(s-2)^2 - 5^2} + \frac{s+3}{(s+3)^2 + 6^2}\right]$.

6. Using Laplace transform solve the differential equation $y'' + 4y' + 3y = e^{-t}$ given that $y(0) = 1$; $y'(0) = 0$.

7. Solve : $z = px + qy + 2\sqrt{pq}$.

8. Solve : $(D+1)^2 = e^{-x} \cos x$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Solve : $y = zpz + y^2 p^3$.

10. Solve : $x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} - 5y = \cos(\log x)$.

11. Solve : $(mz - ny)dx + (nx - lz)dy + (ly - mx)dz = 0$.

12. Use Charpit method to solve $p^2 + q^2 = npq$.

13. (a) Find $L\left[\frac{1-e^t}{t}\right]$ (b) $L\left[\frac{\cos at}{t}\right]$ does exit. (8 + 2)

14. Find $L^{-1}\left[\frac{s+3}{(s^2+6s+13)^2}\right]$.

15. Solve $z = px + qy + p^2 q^2$.

16. Solve $\frac{d^2 y}{dx^2} + y = \sec x$ by method of variation of parameters.