UG–328	**BMS–11/ BMC – 11**

B.Sc. DEGREE EXAMINATION —
DECEMBER, 2019.

First Year

Mathematics

ELEMENTS OF CALCULUS

Time : 3 hours   Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Find the n^{th} differential co–efficient of $y = \sin^3 x$.

2. Find the radius of curvature for the curve $\sqrt{x} + \sqrt{y} = 1$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$.

3. Evaluate : $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 s} dx$.

4. Evaluate : $\lim_{n \to \infty} \frac{n^2}{(n-7)^2 - 6}$

5. (a) If $0 < x < 1$, then show that $\sum_{n=0}^{\infty} x^n$ converges to $\frac{1}{1-x}$.

(b) If $x \geq 1$, then show that $\sum_{n=0}^{\infty} x^n$ diverges.

6. Prove that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

7. Evaluate $\int_0^1 x^7 (1-x)^8 \, dx$.

8. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If $y = \sin^{-1} x$, prove that $(1-x^2) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0$.

10. Find an evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

11. If $I_n = \int_0^{\pi/2} x^n \cos x \, dx$, then show that $I_n + n(n-1)I_{n-2} = \left[\frac{\pi}{2}\right]^n$.

12. If $\{s_n\}_{n=1}^{\infty}$ is a cauchy sequence of real numbers, then show that $\{s_n\}_{n=1}^{\infty}$ is bounded.

13. State and prove Ratio test.

14. Prove that $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.

15. Discuss the maxima and minima of the function $x^3 y^2 (6-x-y)$.

16. Change the order of integration $\int_0^a \int_{x^2/a}^{2a-x} xy \, dx \, dy$ and evaluate it.

UG–330	**BMS–13/ BPHY–01**

B.Sc. DEGREE EXAMINATION — DECEMBER, 2019.

First Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Solve : $xyp^2 + (x + y)p + 1 = 0$

2. Solve : $(D^2 + 4)y = x^2$.

3. Test for exactness and hence solve $(x^2 - 2xy + 3y^2)dx + (y^2 + 6xy - x^2)dy = 0$.

4. Solve the following PDE by Charpit's method $p^2 - xp - q = 0$.

5. Evaluate : $L[te^{2t}\cos 2t]$.

6. Solve : $xp^2 - yp - x = 0$.

7. Solve : $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x \cos x$.

8. Evaluate $L^{-1}\left[\frac{7s-1}{(s+1)(s+2)(s+3)}\right]$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Solve : $p^2 + 2yp\cot x - y^2 = 0$.

10. Solve by variation of parameter method $\frac{d^2y}{dx^2} + 4y = \tan 2x$.

11. Solve : $(mz - ny)\,dx + (nx - lz)\,dy + (lx - my)\,dz = 0$.

12. Find complete and singular solution of $z = px + qy + p^2q^2$.

13. Using Laplace transform solve $\frac{d^2y}{dt^2} + \frac{6dy}{dt} + 5y = e^{-2t}$.

14. Solve : $x^2 \dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = x^2 + 2\log x$.

15. Solve : $(y-z)\,p + (z-x)q = x - y$.

16. Find :

(a) $L^{-1}\left[\log\left(\dfrac{s^2+9}{s^2+1}\right)\right]$

(b) $L\left[\dfrac{\cos 3t - \cos 2t}{t}\right]$.

UG–329 BMS–12/BMC–12

B.Sc. DEGREE EXAMINATION –
DECEMBER, 2019.

First Year

Maths

TRIGONOMETRY, ANALYTICAL GEOMETRY
AND VECTOR CALCULUS

Time : 3 hours | Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that

$$\frac{\sin 7\theta}{\sin\theta} = 64\cos^6\theta - 80\cos^4\theta + 24\cos^2\theta - 1.$$

2. Find the image of the point (1, –2, 3) in the plane $2x - 3y + 2z + 3 = 0$.

3. Find the equation of the sphere passing through the four points $(2, 3, 1)$, $(5, -1, 2)$, $(4, 3, -1)$ and $(2, 5, 3)$.

4. Find the constants a, b, c so that the vector

$$\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$$

is irrotational.

5. Evaluate $\iiint_V \Delta \cdot \vec{F} dv$ if $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and if V is the volume of the region enclose by the cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.

6. If $\sin(A + iB) = x + iy$, prove that

(a) $\dfrac{x^2}{\cosh^2 B} + \dfrac{y^2}{\sinh^2 B} = 1$

(b) $\dfrac{x^2}{\sin^2 A} - \dfrac{y^2}{\cos^2 A} = 1$

7. Find the symmetrical form of the equations of the lines $x + 5y - z - 7 = 0$, $2x - 5y + 3z + 1 = 0$.

8. If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, evaluate $\int_c \vec{F} \cdot d\vec{r}$, where c is the curve on the xy plane $y = 2x^2$ from (0, 0) to (1, 2).

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Sum to infinity the series

$$\cos\alpha + \frac{1}{2}\cos(\alpha+\beta) + \frac{1.3}{2.4}\cos(\alpha+2\beta) + \ldots.$$

10. Prove that the lines

$$\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}; \frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$$

are co-planar. Find also their point of entersection and the plane through them.

11. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0$, $2x - y + 2z = 5$ for a great circle.

12. Find the directional derivative of $xyz - xy^2z^3$ at the point (1, 2, –1) in the direction of the vector $\vec{i} - \vec{j} - 3\vec{k}$.

13. Verify Green's theorem in the plane for $\int (xy - x^2)\,dx + x^2y\,dy$, where C is the boundary of the region bounded by $y = 0$, $x = 1$, $y = x$.

14. Prove that

$$\sin^3\theta\cos^5\theta = \frac{-1}{2^7}(\sin 8\theta + 2\sin 6\theta - 2\sin 4\theta - 6\sin 2\theta)$$

15. Find the perpendicular distance from (3, 9, –1) to the line $\frac{x+8}{-8} = \frac{y-31}{1} = \frac{z-13}{5}$.

16. Evaluate $\iint_s \vec{F}\cdot\hat{n}ds$, where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and s is the surface of the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$.

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