TAMIL NADU OPEN UNIVERSITY

Chennai - 15

School of Science

ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-25, Topology and Functional Analysis
Batch : AY 2018-19 – II Year
No.of Assignment : One Assignment for Each 2 Credits
Maximum CIA Marks : 25 (Average of Total No. of Assignments)

Assignment – I

. Answer any one of the question not exceeding 1000 words. Max : 25 Marks

1. State and Prove Urysohn's lemma.
2. State and Prove Tietze extension theorem.
3. State and Prove Uryshon's metrization theorem.

Assignment – II

Answer any one of the question not exceeding 1000 words.

1. State and Prove Hahn-Banach Theorem, proving necessary result.
2. Prove that If N is normed linear space, then the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak* topology.
3. State and prove Open Mapping theorem, proving necessary result.

Assignment – III

Answer any one of the question not exceeding 1000 words.

1. State and prove Closed Graph theorem, proving necessary result.

2. State and prove Bessel's inequality.

3. If $\{e_i\}$ is an orthonormal set in a Hilbert space H and x is an arbitrary vector in H, then prove that

   $[x - \sum(x, e_i)e_i] \perp e_j$ for each j.

Assignment – IV

Answer any one of the question not exceeding 1000 words.

1. Let H be a Hilbert space and let $\{e_i\}$ be an orthonormal set in H. Then the following conditions are equivalent to one another
   (i) $\{e_i\}$ is complete.
   (ii) $x \perp e_i \ \forall i$ implies x = 0.
   (iii) If x is an arbitrary vector in H, then $x = \sum(x, e_i)e_i$.
   (iv) If x is an arbitrary vector in H, then $||x||^2 = \sum|(x, e_i)|^2$.

2. State and prove Riesz Representation Theorem.

3. If T is a positive operator on a Hilbert space H, then prove that I + T is non-singular.

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ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-26, Operation Research
Batch : AY 2018-19 – II year
No.of Assignment : One Assignment for Each 2 Credits
Maximum CIA Marks : 25 (Average of Total No. of Assignments)

Assignment – I

Answer any one of the question not exceeding 1000 words.

1. Use Simplex method to solve the following L.P.P.
   Maximize $Z = 7x_1 + 5x_2$ subject to the constraints
   $x_1 + 2x_2 \leq 6$, $4x_1 + 3x_2 \leq 12$, $x_1, x_2 \geq 0$

2. Use two-phase simplex method to minimize $Z = \frac{15}{2} x_1 - 3x_2$
   Subject to the constraints $3x_1 - x_2 - x_3 \geq 3$, $x_1 - x_2 - x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$.

3. Use dual simplex method to solve the L.P.P.
   Minimize $Z = x_1 + 2x_2 + 3x_3$ subject to the constraints
   $x_1 - x_2 + x_3 \geq 4$, $x_1 + x_2 + 2x_3 \leq 8$, $x_2 - x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$.

Assignment – II

Answer any one of the question not exceeding 1000 words.

1. Illustrate interior – point algorithm.
2. Apply minimum Spanning Tree algorithm to obtain solution to the Seervada Park problem.
3. Explain the Augmenting Path Algorithm to the Seervada Park problem.

Assignment – III

Answer any one of the question not exceeding 1000 words.

1. Solve the following 3 x 3 game by linear programming

$$\text{Player A} \overset{\text{Player B}}{\begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix}}$$

2. Find the optimum integer solution to the following all I.P.P.
   Maximize $Z = x_1 + 2x_2$
   Subject to the constraints
   $x_1 + x_2 \leq 7$, $2x_1 \leq 11$, $2x_2 \leq 7$, $x_1, x_2 \geq 0$ and are integers.

3. Use Branch-and Bound technique to solve the following I.P.P.

   Maximize $Z = 7x_1 + 9x_2$

   Subject to the constraints

   $-x_1 + 3x_2 \leq 6$, $7x_1 + x_2 \leq 35$, $0 \leq x_1, x_2 \leq 7$, x_1, x_2 are integers.

Assignment – IV

Answer any one of the question not exceeding 1000 words.

1. Explain Pure Birth and Death Model.
2. Explain Specialized Poisson Queues – M/A/1 Queue.
3. Write a note on Separable Convex Programming.

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ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-27, Graph Theory and Algorithms
Batch : AY 2018-19 – II Year
No.of Assignment : One Assignment for Each 2 Credits
Maximum CIA Marks : 25 (Average of Total No. of Assignments)

Assignment – I

Answer any one of the question not exceeding 1000 words.

1. State and prove Menger's Theorem.
2. Write a note on "Operations of Graphs".
3. State equivalent conditions for a graph to be a Tree and Prove.

Assignment – II

Answer any one of the question not exceeding 1000 words.

1. Define an Eulerian Graph, give example and a counter example. Also state and prove a necessary and sufficient condition for a connected graph to be Eulerian.
2. Explain Marriage Problem, proving Hall's Marriage Theorem.
3. State and prove Tutte's Theorem.

Assignment – III

Answer any one of the question not exceeding 1000 words.

1. Explain Stable Matching and prove that for every assignment of preferences in a bipartite graph, there is a stable matching.
2. State and prove Vizing's Theorem.
3. State and prove Brooks Theorem.

Assignment – IV

Answer any one of the question not exceeding 1000 words.

1. Write a note on Mycielski's Construction.
2. Define the chromatic polynomial of a graph and list some properties of chromatic polynomial and justify your answer.
3. Prove the following are equivalent:

   a) Every planner graph is 4-vertex colourable.

   b) Every plane graph is 4-face colourable.

   c) Every simple 2-edge connected 3-regular planar graph is 3-edge colourable.

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ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-28, Differential Equations
Batch : AY 2018-19 – II Year
No.of Assignment : One Assignment for Each 2 Credits
Maximum CIA Marks : 25 (Average of Total No. of Assignments)

Assignment – I

Answer any one of the question not exceeding 1000 words.

1. State and prove the Existence Theorem for the initial value problem of a second order linear homogeneous differential equation.

2. Let $\emptyset_1, \emptyset_2, \ldots, \emptyset_n$ are n solutions of a linear differential equation L(y) = 0 on an interval I. Then prove that $\emptyset_1, \emptyset_2, \ldots, \emptyset_n$ are independent on I if and only if $W(\emptyset_1, \emptyset_2, \ldots, \emptyset_n)(x) \neq 0$ for all x in I.

3. Let $\{\emptyset_1, \emptyset_2, \ldots, \emptyset_n\}$ be *n* solutions of a linear homogeneous equation with constant coefficients L(y) = 0 on an interval I containing a point x_0. Then prove that $W(\emptyset_1, \emptyset_2, \ldots, \emptyset_n)(x) = e^{-a1(x-x0)} W(\emptyset_1, \emptyset_2, \ldots, \emptyset_n)(x_0)$.

Assignment – II

Answer any one of the question not exceeding 1000 words.

1. Explain the Algorithm of variation of Parameter method and also find a particular solution of $y'' + y = \text{cosec } x$.

2. Find the solution of the initial value problem $y'' - 2y' + y = 2x, y(0) = 6, y'(0) = 2$.

3. Prove that for each n there is one and only one polynomial solution $P_n(x)$ of degree n for the Legendre equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ satisfying $P_n(1) = 1$.

Assignment – III

Answer any one of the question not exceeding 1000 words.

1. Define Bessel Equation and solve.
2. Solve the initial value problem $y'' - 2y' + y = 0$, $y(0) = 0$, $y'(0) = 1$ on the interval $[0,a]$ where a is any positive real number.
3. Find a fundamental matrix of the equation

$$y' = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix} y$$

Assignment – IV

Answer any one of the question not exceeding 1000 words.

1. State and prove Picard's Theorem.
2. Reduce the equation into its canonical form: $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$.
3. Write a note on Characteristic Curves.