1. Prove that the set \( \mathbb{R} \) of real numbers is uncountable.

2. A function \( f \) is defined on \( \mathbb{R} \) by
\[
f(x) = \begin{cases} 
-x^2, & x \leq 0 \\
5x - 4, & 0 < x < 1
\end{cases}
\]
Examine \( f \) for continuity at \( x = 0 \).

3. Prove that every compact subset of a metric space is closed.

4. State and prove Rolle’s theorem.
5. Find the analytic function \( f(z) \), if 
\[ u = x^3 - 3xy^2 + 3x^2y - 3y^2 + 1. \]

6. Define conformal mapping and discuss the transformation \( w = z + c \), where \( c \) is a complex constant.

7. Find the residue of \( f(z) = \frac{z^2}{(z - 1)(z + 2)^2} \) at its singularities.

8. Find the Laurent’s series of \( f(z) = \frac{1}{z(1-z)} \) valid in the region \( |z + 1| < 1 \).

PART B — (5 \times 10 = 50 marks)

Answer any FIVE questions.

9. State and prove Holder’s inequality.

10. Show that continuous image of a connected space is connected.

11. Show that the function \( f(x) = \frac{1}{x} \) is not uniformly continuous on \((0, 1]\).
12. Let \((X, d)\) be any metric space. A subset \(F\) of \(X\) is closed if and only if its complement in \(X\) is open.


14. Find the bilinear transformation which maps the points 1, -1, 1 of the \(z\) — plane into the points 0, 1, \(\infty\) of the \(w\) — plane respectively.

15. State and prove Cauchy’s residue theorem.

16. Evaluate \(\int_{-\infty}^{\infty} \frac{x^2 \, dx}{(x^2 + 25)(x^2 + 9)}\), using contour Integration.
B.Sc. DEGREE EXAMINATION —
DECEMBER, 2018.

Third Year

Mathematics with Computer Applications

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Show that a nonempty subset \( S \) of a vector space \( V \) is a subspace of \( V \) if and only if \( S \) is closed with respect to vector addition and scalar multiplication in \( V \).

2. Prove that any subset of a linearly independent set is linearly independent.

3. Prove that \( S = \{(1, 0, 0), (0, 1, 0), (1, 1, 1), (1, 1, 0)\} \) spans the vector space \( V_3(\mathbb{R}) \) but is not a basis.
4. If $W$ is a subspace of a finite dimensional, inner product space $V$. Show that $V = W \oplus W^\perp$.

5. State and prove Schwartz inequality.

6. Show that a bilinear form $f$ defined on $V$ is symmetric if and only if its matrix $(a_{ij})$ with respect to any one basis $\{v_1, v_2, \ldots, v_n\}$ is symmetric.

7. Show that in a poset, the least element and the greatest element are unique if it exists.

8. Define modular lattice and show that every distributive lattice is modular.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let $V$ and $W$ be vector spaces over field $F$. Let $L(V, W)$ represent the set of all linear transformation from $V$ to $W$. Prove that $L(V, W)$ itself is a vector space over $F$ under addition and scalar multiplication defined by $(f + g)(v) = f(v) + g(v)$ and $(\alpha f)(v) = \alpha f(v)$.
10. Prove that any vector space of dimensions $n$ over a field $F$ is isomorphic to $V_n(F)$.

11. Let $V$ be a finite dimensional vector space over a field $F$. Let $W$ be a subspace of $V$. Then show that
$$\dim \frac{V}{W} = \dim V - \dim W.$$ 

12. Show that every finite dimensional inner product space has an orthonormal basis.

13. Reduce the quadratic form
$$2x_1x_2 - x_1x_3 + x_1x_4 - x_2x_3 + x_2x_4 - 2x_3x_4$$
to the diagonal form using Lagrange’s method.

14. (a) Find a vector of unit length which is orthogonal to $(1, 3, 4)$ in $V_3(R)$ with standard inner product.
   (b) Find an orthogonal basis containing the vector $(1, 3, 4)$ for $V_3(R)$ with standard inner product.

15. Let $f$ be the bilinear form defined on $V_2(R)$ by
$$f(x, y) = x_1y_1 + x_2y_2$$
where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of $f$,
   (a) with respect to the standard basis $\{e_1, e_2\}$
   (b) with respect to the basis $\{(1, 1), (1, 2)\}$
16. (a) Show that in any distributive lattice

\[(a \lor b) \land (b \lor c) \land (c \lor a) = (a \land b) \lor (b \land c) \lor (c \land a)\]

(b) Let \( B \) be a Boolean algebra. Show that

\[(a \lor b)' = a' \land b', (a \land b)' = a' \lor b' \text{ and } (a')' = a.\]
B.Sc. DEGREE EXAMINATION — DECEMBER, 2018.

Third Year

Mathematics with Computer Applications

LINEAR PROGRAMMING AND OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

SECTION A — (5 x 5 = 25 marks)

Answer any FIVE questions.

1. Explain Big-M method of solving linear programming problems.

2. Explain slack and surplus variable.

3. Explain North-West corner rule of Transportation problem.

4. Define assignment problem.

5. Explain game theory.
6. Mention some of the advantages and disadvantages of having inventory.

7. Explain
   (a) Shortage cost
   (b) Carrying cost.

8. Explain queue discipline.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.


10. Solve the following L.P.P. by simplex method.
    Max. \( z = 5x_1 + 3x_2 \)
    subject to
    \[ 3x_1 + 5x_2 \leq 15 \]
    \[ 5x_1 + 2x_2 \leq 10 \]
    \[ x_1, x_2 \geq 0 \]

11. Solve the following transportation problem

    | A  | B  | C  | Availability |
    |----|----|----|--------------|
    | Source 1 | 6 | 8 | 4 | 14 |
    | Source 2 | 4 | 9 | 3 | 12 |
    | Source 3 | 1 | 2 | 6 | 5 |
    | Required 6 | 10 | 15 |

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13. Solve the game using graphical method

\[
\begin{array}{ccc}
 & I & II & III & IV \\
I & 1 & 4 & -2 & -3 \\
II & 2 & 1 & 4 & 5 \\
\end{array}
\]

Player A

14. Explain the queueing model \((M/M/1):(\infty/:FCFS)\)

15. Explain EOQ model – with or without shortages and multi item inventory model with constraints.

16. A T.V. Repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in, and it the arrival of sets is approximately Poisson with an average of 10 per 8 hours day, what is the repairman’s expected idle time each day? How many jobs are seed of average set just brought in?
B.Sc. DEGREE EXAMINATION —
DECEMBER 2018.

Third Year

MATHEMATICS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Draw a connected graph with all vertices having even degree.

2. Show that the sum of the degrees of the points of a graph G is twice as the number of lines.

3. Check whether the sequence (3, 3, 2, 2, 2, 2, 1) is graphic or not.

4. Show that every connected graph G contains a spanning tree.

5. Prove that a connected graph G is Eulerian if all the vertices of G are of even degree.
6. Define edge coloring and vertex coloring of a graph.

7. Prove that every planar graph is 6-vertex colourable.

8. State and prove Euler formula for planar graphs.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE of the following.

9. (a) Show that the number of odd degree vertices in a graph G is even.

(b) Show that a connected \((p,q)\) - graph contains a cycle \(q \geq p\).

10. Prove that a graph G is connected if and only if for every partition of \(V\) into two non-empty sets \(V_1\) and \(V_2\) there is an edge with one end in \(V_1\) and the other end is \(V_2\).

11. Show that every nontrivial graph contains at least two vertices which are not cut vertices.

12. Show that a nontrivial connected graph is Eulerian if and only if it has no vertex of odd degree.
13. If $G$ is a graph with $p \geq 3$ and, then prove that $G$ is Hamiltonian.

14. Find the chromatic polynomial of a tree of order $p$.

15. Prove that every tournament has a directed Hamilton path.

16. If $G$ is a bipartite graph with $q(G) \geq 1$ then show that $\chi_1(G) = \Delta(G)$.
B.Sc. DEGREE EXAMINATION —
DECEMBER 2018.

Third Year

Mathematics with Computer Applications

INTRODUCTION TO INTERNET
PROGRAMMING (JAVA)

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Discuss about Java constants with example.

2. What are constructors? Discuss about the usage of constructors with examples.

3. Write short notes on : Common Java Exceptions.

4. Write a Java program to illustrate the use of Flow Layout Manager.

5. Write a program in Java to print the row sum of each row in a matrix.
6. Write a program to read $n$ integers and print the odd numbers in them.

7. Why java is Object – Oriented language? Discuss in Detail.

8. Explain about applet life cycle.

PART B — (5 x 10 = 50 marks)

Answer any FIVE of the following.

9. Discuss briefly on Data Types in Java with examples.

10. Explain branching and looping statements in Java with suitable examples.

11. Write in detail about wrapper classes in Java.

12. Explain about creating packages and accessing a package with examples.

13. Write an Applet program to draw a computer using the paint method.
14. Discuss in detail about inheritance in java.

15. Discuss in detail, defining extending and implementing interface with example.

16. Discuss in detail about Java features.