B.Sc. DEGREE EXAMINATION —
JUNE 2018.
Third Year
Mathematics
REAL AND COMPLEX ANALYSIS

Time : 3 hours  Maximum marks : 75

PART A — (5 × 5 = 25 marks)
Answer any FIVE questions.

1. Prove that the set \( \{x : 0 \leq x \leq 1\} \) is uncountable.

2. If \((M, \rho)\) is a complete metric space and \(A\) is a closed subset of \(M\), then prove that \((A, \rho)\) is also complete.

3. Let \(f\) be a continuous function from a metric space \(M_1\) into a metric space \(M_2\). If \(M_1\) is connected then prove that the range of \(f\) is also connected.
4. Let $f$ be a continuous real-valued function on the closed bounded interval $[a,b]$. If the maximum value of $f$ is attained at $c$ where $a < c < b$ and if $f'(c)$ exists then prove that $f'(c) = 0$.

5. Prove that $\sqrt{x}$ is a continuous function on $[0, \infty)$.

6. Find the value of $a, b, c, d$ so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ may be analytic.

7. Evaluate $\int_c \frac{z+1}{(z^2 - 2z^2)} \, dz$ where $c$ is the circle $|z - 2 - i| = 2$ by using Cauchy's integral formula.

8. Find the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the region $1+|z+1|<2$.

PART B — (5 x 10 = 50 marks)

Answer any FIVE questions.

9. Let $l^\infty$ denote the set of all bounded sequence of real numbers. If $x = \{x_n\}_{n=1}^\infty$ and $y = \{y_n\}_{n=1}^\infty$ are points in $l^\infty$, define $\rho(x,y) = \lim_{1 \leq n < \infty} |x_n - y_n|$ then prove that $(l^\infty, \rho)$ is a metric space.
10. The metric space \((M, \rho)\) is compact if and only if every sequence of points in \(M\) has a sequence converging to a point in \(M\).

11. State and prove the second fundamental theorem of calculus.

12. State and prove Taylor's formula with the integral form of the remainder.

13. State and prove a sufficient condition for differentiability of complex-valued functions.

14. Find the bilinear transformation which maps the points \(0, -i, -1\) of the \(z\)-plane into the points \(i, 1, 0\) of the \(w\)-plane respectively.

15. State and prove Rouche's theorem.

16. Evaluate \(\int_{0}^{2\pi} \frac{d\theta}{a + b \cos \theta}\), using contour integration, where \(a > b > 0\).
B.Sc. DEGREE EXAMINATION —
JUNE 2018.

Third Year

Mathematics

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let \( R^+ \) be the set of all positive real numbers. Define addition and scalar multiplication as follows:

(a) \( u + v = uv \) for all \( u, v \in R^+ \)

(b) \( au = u^a \) for all \( u \in R^+ \) and \( a \in R \).

Prove that \( R^+ \) is a real vector space.

2. Show that the mapping \( T : V_2(R) \rightarrow V_3(R) \) defined by \( T(a, b) = (a + b, a - b, b) \) is a linear transformation.
3. Let V be a vector space over a field F. Show that any subsets of V containing the zero vector is linearly independent.

4. Show that every linearly independent subset of a finite dimensional vector space V forms a part of a basis.

5. Let $S = \{v_1, v_2, \ldots, v_n\}$ be an orthogonal set of non-zero vectors in an inner product space V. Show that $S$ is linearly independent.

6. Let $f$ be the bilinear form defined on $V_2(R)$ by $f(x, y) = x_1y_1 + x_2y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of $f$ with respect to the standard basis $\{e_1, e_2\}$.

7. Define partial order relation on a set and give an example. Further check whether the relation 'a divides b' is a partial order on the set $Z$ of integers.

8. Prove that any distributive lattice $L$ is a modular lattice.

PART B — (5 x 10 = 50 marks)

Answer any FIVE questions.

9. State and prove fundamental theorem of vector space homomorphism.
10. Let $V$ be a vector space over a field $F$. Let $S, T \subseteq V$. Prove that
   (a) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$
   (b) $L(SUT) = L(S) + L(T)$

11. Let $V$ and $W$ be vector spaces over a field $F$. Let $T : V \to W$ be an isomorphism. Prove that $T$ maps a basis of $V$ onto a basis of $W$.

12. Let $V$ and $W$ be two finite dimensional vector space over a field $F$. Let $\dim V = m$ and $\dim W = n$. Show that $L(V, W)$ is a vector space of dimension $mn$ over $F$.

13. Apply Gram—Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$. where $v_1 = \{1, 0, 1\}$, $v_2 = \{1, 3, 1\}$ and $v_3 = \{3, 2, 1\}$.

14. Show that the set of all bilinear forms on a vector space $V$ is also a vector space over $F$. 

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15. Reduce the quadratic form
\[ x_1^2 + 4x_1x_2 + 4x_1x_3 + 4x_2^2 + 16x_2x_3 + 4x_3^2 \]
to the diagonal form.

16. (a) Let B be a Boolean algebra. Show that
\[(a \lor b)' = a' \land b', (a \land b)' = a' \lor b' \text{ and } (a')' = a.\]

(b) In a Boolean algebra if \( a \lor x = b \lor x \) and \( a \lor x' = b \lor x' \) Show that \( a = b \).
1. Write basic assumptions in Linear Programming Models.

2. Write the dual of problem
   \[ \text{Max } Z = 2x_1 + 3x_2 + x_3 \]
   subject to \( 4x_1 + 3x_2 + x_3 = 6 \)
   \( x_1 + 2x_2 + 5x_3 = 4 \)
   \( x_1, x_2, x_3 \geq 0 \)
3. Define assignment problem and mention the necessary basic steps to solve it.

4. Find an initial basic feasible solution of the following transportation problem.

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>O2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>O3</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>9</td>
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<td></td>
<td>20</td>
<td>40</td>
<td>30</td>
<td>10</td>
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</table>

5. Solve the game whose pay off matrix is

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-3</td>
<td>-2</td>
<td>6</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>III</td>
<td>5</td>
<td>-2</td>
<td>4</td>
</tr>
</tbody>
</table>

6. Mention some of the advantages and disadvantages of having inventory.

7. Explain
   (a) Shortage Cost
   (b) Carrying Cost.

8. Explain Queue discipline.
9. Solve the following Linear programming problem by Simplex method.

\[ \text{Max } Z = 5x_1 + 3x_2 \]

subject to \[ 3x_1 + 5x_2 \leq 15 \]
\[ 5x_1 + 2x_2 \leq 10 \]
\[ x_1, x_2 \geq 0. \]

10. Explain the concept of duality.

11. Solve the following transportation problem.

<table>
<thead>
<tr>
<th>Destination</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>8</td>
<td>8</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>11</td>
<td>9</td>
<td>7</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>9</td>
<td>7</td>
<td>13</td>
<td>50</td>
</tr>
<tr>
<td>Demand</td>
<td>35</td>
<td>28</td>
<td>35</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

12. Solve the following assignment problem

<table>
<thead>
<tr>
<th>J1</th>
<th>J2</th>
<th>J3</th>
<th>J4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>B</td>
<td>16</td>
<td>20</td>
<td>28</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>18</td>
<td>30</td>
</tr>
<tr>
<td>D</td>
<td>9</td>
<td>24</td>
<td>32</td>
</tr>
</tbody>
</table>
13. Solve the game by graphical method

\[
\begin{array}{cccc}
\text{I} & \text{II} & \text{III} & \text{IV} \\
\text{A} & 1 & 4 & -2 & 3 \\
\text{II} & 2 & 1 & 4 & 5 \\
\end{array}
\]

14. Explain basic classification of characteristics of Inventory systems.

15. Explain the queuing model \((M/M/1):(\infty/FCFS)\).

16. Prove that if arrival occur at random in time, the number of arrivals occurring in a fixed time interval follows a poisson distribution.
B.Sc. DEGREE EXAMINATION —
JUNE, 2018.

Third Year
Mathematics With Computer Application

GRAPH THEORY

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)
Answer any FIVE questions.

1. When do you say that two graphs are isomorphic? Give an example.
2. Draw a connected graph with all vertices having even degree.
3. Define a bipartite graph and give an example.
4. Define connectivity and edge-connectivity of a graph and illustrate with examples.
5. Write a note on Hamiltonian graphs.
6. Define a chromatic number of a graph.
7. Prove that a graph $K_{3,3}$ is non-planar.
8. Write a short note on Tournaments.
SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that a graph G is connected if and only if for every partition of V into two non-empty sets \( V_1 \) and \( V_2 \), there is an edge with one end in \( V_1 \) and one end in \( V_2 \).

10. Prove that a graph is bipartite if and only if it contains no odd cycle.

11. Explain with illustrations degree sequence and graphic sequence.

12. Prove that a connected graph has an Euler trial if and only if it has at most two vertices of odd degree.

13. If G is a graph with \( p \geq 3 \) and \( \delta \geq \frac{p}{2} \), then prove that G is Hamiltonian.

14. If G is bipartite, then prove that \( \chi_1(G) = \Delta(G) \).

15. Prove that a non-empty G is 2-colourable if and only if G is bipartite.

16. Prove if G is a connected plane graph then \( p - q + r = 2 \).
B.Sc. DEGREE EXAMINATION —
JUNE 2018.

First Year

Mathematics with Computer Applications

INTRODUCTION TO INTERNET
PROGRAMMING (JAVA)

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Discuss the general structure of a Java program.

2. Specify the general form of switch statement in Java and explain with examples.

3. Write a Java program to convert the given temperature in Fahrenheit to Celsius using the conversion formula $C = \frac{F - 32}{1.8}$.
4. Write a program to compute the sum of the digits of a given integer number.

5. How do we create an one-dimensional array? Explain with example.

6. Describe various forms of implementing interfaces.

7. What is a thread? Write the difference between multithreading and multitasking.

8. Describe the various sections of a web page.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE of the following.

9. Discuss different types of constants that are available in Java.

10. Explain if, else if ladder and nested if constructs in Java.

11. What is a constructor? What are its special properties? How do you invoke a constructor?

12. Write a java program to solve a quadratic equation completely.

13. Describe different forms of inheritance with examples.
14. Explain how exception handing mechanism can be used for debugging a program.

15. What is a package? How do you design a package?

16. Develop an applet that receives three numeric values as input from the user and then displays the largest of the three on the screen. Write a HTML page to test the applet.