1. Prove that the set \( \mathbb{R} \) of real numbers is uncountable.

2. A function \( f \) is defined on \( \mathbb{R} \) by

\[
  f(x) = \begin{cases} 
  -x^2, & x \leq 0 \\
  5x - 4, & 0 < x < 1
  \end{cases}
\]

Examine \( f \) for continuity at \( x = 0 \).

3. Prove that every compact subset of a metric space is closed.

4. State and prove Rolle’s theorem.
5. Find the analytic function $f(z)$, if $u = x^3 - 3xy^2 + 3x^2y - 3y^2 + 1$.

6. Define conformal mapping and discuss the transformation $w = z + c$, where $c$ is a complex constant.

7. Find the residue of $f(z) = \frac{z^2}{(z-1)(z+2)^2}$ at its singularities.

8. Find the Laurent’s series of $f(z) = \frac{1}{z(1-z)}$ valid in the region $|z+1| < 1$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. State and prove Holder’s inequality.

10. Show that continuous image of a connected space is connected.

11. Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0, 1]$.
12. Let \((X, d)\) be any metric space. A subset \(F\) of \(X\) is closed if and only if its complement in \(X\) is open.


14. Find the bilinear transformation which maps the points 1, -1, 1 of the \(z\) — plane into the points 0, 1, \(\infty\) of the \(w\) — plane respectively.

15. State and prove Cauchy’s residue theorem.

16. Evaluate \(\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 25)(x^2 + 9)} \, dx\), using contour Integration.
Show that a nonempty subset $S$ of a vector space $V$ is a subspace of $V$ if and only if $S$ is closed with respect to vector addition and scalar multiplication in $V$.

Prove that any subset of a linearly independent set is linearly independent.

Prove that $S = \{(1, 0, 0), (0, 1, 0), (1, 1, 1), (1, 1, 0)\}$ spans the vector space $V_3(\mathbb{R})$ but is not a basis.
4. If $W$ is a subspace of a finite dimensional, inner product space $V$. Show that $V = W \oplus W^T$.

5. State and prove Schwartz inequality.

6. Show that a bilinear form $f$ defined on $V$ is symmetric if and only if its matrix $(a_{ij})$ with respect to any one basis $\{v_1, v_2, \ldots, v_n\}$ is symmetric.

7. Show that in a poset, the least element and the greatest element are unique if it exists.

8. Define modular lattice and show that every distributive lattice is modular.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let $V$ and $W$ be vector spaces over field $F$. Let $L(V, W)$ represent the set of all linear transformation from $V$ to $W$. Prove that $L(V, W)$ itself is a vector space over $F$ under addition and scalar multiplication defined by $(f + g)(v) = f(v) + g(v)$ and $(\alpha f)(v) = \alpha f(v)$.
10. Prove that any vector space of dimensions \( n \) over a field \( F \) is isomorphic to \( V_n(F) \).

11. Let \( V \) be a finite dimensional vector space over a field \( F \). Let \( W \) be a subspace of \( V \). Then show that 
\[
\dim \frac{V}{W} = \dim V - \dim W.
\]

12. Show that every finite dimensional inner product space has an orthonormal basis.

13. Reduce the quadratic form 
\[
2x_1x_2 - x_1x_3 + x_1x_4 - x_2x_3 + x_2x_4 - 2x_3x_4
\]

14. (a) Find a vector of unit length which is orthogonal to \((1, 3, 4)\) in \( V_3(R) \) with standard inner product.

(b) Find an orthogonal basis containing the vector \((1, 3, 4)\) for \( V_3(R) \) with standard inner product.

15. Let \( f \) be the bilinear form defined on \( V_2(R) \) by 
\[
f(x,y) = x_1y_1 + x_2y_2 \quad \text{where} \quad x = (x_1, x_2) \quad \text{and} \quad y = (y_1, y_2).
\]
Find the matrix of \( f \),

(a) with respect to the standard basis \( \{e_1, e_2\} \)

(b) with respect to the basis \( \{(1, 1), (1, 2)\} \)
16.  (a) Show that in any distributive lattice

\[(a \lor b) \land (b \lor c) \land (c \lor a) = (a \land b) \lor (b \land c) \lor (c \land a)\]

(b) Let \( B \) be a Boolean algebra. Show that

\[(a \lor b)' = a' \land b', (a \land b)' = a' \lor b' \text{ and } (a')' = a.\]
B.Sc. DEGREE EXAMINATION —
DECEMBER, 2018.

Third Year

Mathematics with Computer Applications

LINEAR PROGRAMMING AND OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Explain Big-M method of solving linear programming problems.

2. Explain slack and surplus variable.

3. Explain North-West corner rule of Transporation problem.

4. Define assignment problem.

5. Explain game theory.
6. Mention some of the advantages and disadvantages of having inventory.

7. Explain
   (a) Shortage cost
   (b) Carrying cost.

8. Explain queue discipline.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.


10. Solve the following L.P.P. by simplex method.
    Max. \( z = 5x_1 + 3x_2 \)
    subject to
    \[
    \begin{align*}
    3x_1 + 5x_2 & \leq 15 \\
    5x_1 + 2x_2 & \leq 10 \\
    x_1, x_2 & \geq 0
    \end{align*}
    \]

11. Solve the following transportation problem
    \[
    \begin{array}{cccc|c}
    & A & B & C & \text{Availability} \\
    \text{Source} & 1 & 6 & 8 & 4 & 14 \\
    & 2 & 4 & 9 & 3 & 12 \\
    & 3 & 1 & 2 & 6 & 5 \\
    \text{Required} & 6 & 10 & 15 & \\
    \end{array}
    \]

13. Solve the game using graphical method

\[
\begin{array}{cccc}
  & I & II & III & IV \\
\hline
\text{Player A} & 1 & 4 & -2 & -3 \\
\text{II} & 2 & 1 & 4 & 5 \\
\end{array}
\]

14. Explain the queueing model \( (M/M/1):(\infty/FCFS) \)

15. Explain EOQ model – with or without shortages and multi item inventory model with constraints.

16. A T.V. Repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in, and if the arrival of sets is approximately Poisson with an average of 10 per 8 hours day, what is the repairman’s expected idle time each day? How many jobs are seed of average set just brought in?
B.Sc. DEGREE EXAMINATION —  
DECEMBER 2018.  

Third Year  

OPTIMIZATION TECHNIQUES  

Time : 3 hours Maximum marks : 75  

PART A — (5 × 5 = 25 marks)  

Answer any FIVE questions.  

1. Solve the following LPP by Simplex method.  
   \[ \text{Maximize } Z = 4x + 10y \]  
   subject to  
   \[ 2x + y \leq 50, \]  
   \[ 2x + 5y \leq 100, \]  
   \[ 2x + 3y \leq 90, \text{ and } x, y \geq 0 \]  

2. The assignment cost of assigning anyone operator (I, II, II, IV) to any one machine (A, B, C, D) is given in the following table  

<table>
<thead>
<tr>
<th>Machine /Operators</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>5</td>
<td>13</td>
<td>15</td>
</tr>
</tbody>
</table>
Find the optimal assignment by Hungarian method.

3. Solve the following $2 \times 2$ game

$$
\begin{pmatrix}
B & A \\
3 & 5 & 1 & 3 \\
9 & 1 & 1 & 3
\end{pmatrix}
$$

4. Explain the different costs involved in inventory problems.

5. In a supermarket, the average arrival rate of customer is 10 in every 30 minutes following Poisson process. The average time taken by the cashier to list and calculate the customer’s purchase is 2.5 minutes, following exponential distribution. What is the probability that the queue length exceeds 6?

6. Write down the algorithm of solving integer programming problem by cutting plane method.

7. Find the initial basic feasible solution of the following transporting problem by Vogel Approximation Method.

<table>
<thead>
<tr>
<th>Source/Destination</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>11</td>
<td>13</td>
<td>17</td>
<td>14</td>
<td>250</td>
</tr>
</tbody>
</table>

Find the optimal assignment by Hungarian method.
8. Solve the game whose pay off matrix is given below.

\[
\begin{bmatrix}
9 & 3 & 1 & 8 & 0 \\
6 & 5 & 4 & 6 & 7 \\
2 & 4 & 3 & 3 & 8 \\
5 & 6 & 2 & 2 & 1
\end{bmatrix}
\]

PART B — (5 x 10 = 50 marks)

Answer any FIVE questions.

9. Solve the following LPP by Simplex method.

Maximize \( Z = 2x + 3y \)
subject to \(-x + 2y \leq 4, \)
\(x + y \leq 6, \)
\(x + 3y \leq 9, \) and \(x, y\) are unrestricted
10. Solve the following transportation problem.

Source/Destination D1 D2 D3 D4 Availability

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>11</td>
<td>21</td>
<td>8</td>
</tr>
<tr>
<td>D2</td>
<td>20</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>D3</td>
<td>7</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>D4</td>
<td>8</td>
<td>12</td>
<td>9</td>
</tr>
</tbody>
</table>

50  40  70

Requirements 30 25 35 40

11. Using graphical method, solve the rectangular game whose pay off matrix for player A is

\[
\begin{pmatrix}
  2 & -1 & 5 & -2 & 6 \\
-2 & 4 & -3 & 1 & 0
\end{pmatrix}
\]

12. A particular item has a demand of 9000 units / year. The cost of one procurement is Rs. 100 and the holding cost per unit is Rs 2.40 per year. The replacement is instantaneous and no shortage is allowed. Determine

(a) the economic lot size

(b) the number of orders per year
(c) the between orders

(d) The total cost per year if the cost per unit is Rs. 5.

13. Workers come to tool store room to enquire about specials for accomplishing a particular project assigned to them. The average time between two arrivals is one minute and the arrivals are assumed to be in Poisson process. The average service time of the tool room attendant is 40 seconds. Determine

(a) average queue length.

(b) average number of workers in system

(c) mean waiting time of an arrival in the queue.

(d) Average time that a worker spends in the store room.

14. Find the optimum integer solution to the following LPP.

Maximize \( Z = x + y \)
subject to \( 3x + 2y \leq 5, y \leq 2 \)

\( x, y \) are non-negative integers.
15. Explain any two methods of finding initial basic feasible solution to transportation problem.

B.Sc. DEGREE EXAMINATION –
DECEMBER, 2018.

Third Year

Mathematics with Computer Applications

PROGRAMMING IN C AND C++

Time : 3 hours             Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Describe the purpose of escape sequence characters.

2. Compare while and do .... while loops in C.

3. Write a C-program to calculate simple interest and total amount given P,N,R.

4. Write a C-program to compute factorial of a given number using recursive techniques.

5. What is a pointer? How are the pointer variables declared?
6. Write a short note on
   (a) `fopen()`,
   (b) `fclose()`,
   (c) `feof()` functions in C.

7. Explain basic concepts of object oriented programming.

8. What do you mean by function overloading? Explain.

PART B — (5 × 10 = 50 marks)

Answer any FIVE of the following.

9. Explain various assignment operators, increment and decrement operators with illustration.

10. Explain `if, if ... else, and else if` ladder with suitable examples.

11. Explain the importance of `continue, break and goto` statements in C.

12. Explain the four different storage classes in detail.

13. Write a C-program to arrange a given set of \( n \) numbers ascending order.
14. What is a structure? Discuss its accession, usage, reading and printing with suitable illustrations.

15. A file named DATA contains a series of integers. Code a program to read these numbers and then write all odd numbers to a file to be called ODD and even numbers to a file to be called EVEN.

16. Write a short note on constructors and destructors in C++.
PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. If $G$ is a graph with $\delta(G) \geq n$, then prove that $G$ has a path of length $n$.

2. Define a tree and prove that every non-trivial tree $G$ has at least two vertices of degree 1.

3. Define a Hamiltonian graph and prove that every Hamiltonian graph is 2-connected.

4. If $G$ is uniquely $n$-colourable, show that $\delta(G) \geq n - 1$.

5. If two digraphs are isomorphic, then prove that corresponding points have the same degree pair.
6. Prove that every planar graph $G$ contains a vertex of degree at most 5.

7. Find $x$ if the degree sequence $8, x, 7, 6, 6, 5, 4, 3, 3, 1, 1, 1$ is graphical.

8. Prove that a closed walk of odd length contains a cycle.

PART B — (5 x 10 = 50 marks)

Answer any FIVE questions.

9. Define the operations of union, sum, product and composition of two graphs and illustrate with examples.

10. (a) Prove that any graph with $p$ points and 
    $$\delta \geq \frac{p-1}{2}$$
    is connected.
    
    (b) Prove that line $x$ of a connected graph $G$ is a bridge if and only if $x$ is not on any cycle of $G$.

11. For a connected graph, prove that the following statements are equivalent
    (a) $G$ is Eulerian.
    (b) Every point of $G$ has even degree.
    (c) The set of edges of $G$ can be portioned into cycles.
12. If $G$ is a connected graph having $V, E$ and $F$ as the set of vertices, edges and faces respectively, prove that $|V| - |E| + |F| = 2$, where $|X|$ denotes the cardinality of the set $X$.

13. Prove that the edges of a connected graph $G = (V, E)$ can be oriented so that the resulting digraph is strongly connected iff every edge of $G$ is contained in at least one cycle.

14. Define the closure $c(G)$ of a graph $G$ and prove that it is well defined.

15. (a) If a partition $P = (d_1, d_2, \ldots, d_p)$ with $d_1 \geq d_2 \geq \ldots \geq d_p$ is graphical then prove that

$$
\sum_{i=1}^{p} d_i \text{ is even and } \\
\left( \sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{p} \min\{k, d_i\} \right) \text{ for } 1 \leq k \leq p.
$$

(b) Prove that there is no 3-connected graph with 7 edges.
16. Let $G$ be $(p, q)$ graph. Prove that the following statements are equivalent.

(a) $G$ is a tree

(b) Every two points of $G$ are joined by a unique path.

(c) $G$ is connected and $p = q + 1$.

(d) $G$ is acyclic and $p = q + 1$.

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